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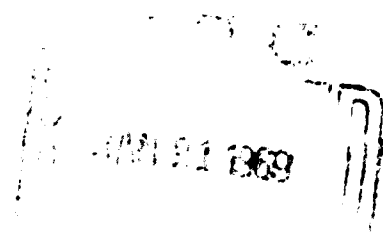


## EQUIVALENT CIRCUITS FOR NEGATIVE RESISTANCE DEVICES

Ross M. Kaplan

TECHNICAL REPORT NO. RADC-TR- 68-356  
December 1968

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## EQUIVALENT CIRCUITS FOR NEGATIVE RESISTANCE DEVICES

Ross M. Kaplan

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## FOREWORD

Research described in this Technical Report was accomplished under System 673A. Distribution of the report is restricted under the provisions of the U.S. Mutual Security Acts of 1949 to protect technology revealed herein.

The author is a native of Utica, New York. He received his Bachelor of Science in Electrical Engineering from Syracuse University in June 1963 and his Master of Science in Electrical Engineering also from Syracuse in January 1968. Sincere appreciation is extended to Professor Bernard Silverman for his guidance and constructive suggestions and to Mrs. Mary Cavalleri for her valuable typing assistance.

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## ABSTRACT

This report examines certain phases of the dynamic behavior of both an open-circuit stable, two-terminal negative-resistance device with its specific equivalent circuit, and a short-circuit stable, two-terminal negative-resistance device with its specific equivalent circuit. The equivalent circuits for these two devices are duals of each other. The static current/voltage characteristic curves for the two types of negative-resistance devices affect their dynamic equivalent circuits.

It is shown that the static current/voltage characteristic curve for the negative-resistance device has a limited negative-slope region since the device possesses particular physical restrictions. The complete equivalent circuit for the current-controlled and voltage-controlled negative-resistance devices is derived by recognizing specific information about the device. The basic equivalent circuit for the negative-resistance device is obtained by assuming a cause-effect relationship between the current and voltage of the device and then by using the Taylor Series expansion on the cause. The basic equivalent circuit for the negative-resistance device is related to the static current/voltage characteristic curve when the device is operated as a trigger circuit. Expressions for the input admittance of the current-controlled negative-resistance device and the input impedance of the voltage-controlled negative-resistance device are derived from the equivalent circuit of the device. The homogeneous linear second-order differential equation that describes the limited behavior of the negative-resistance device is also obtained from the equivalent circuit of the device. The fundamental energy storage element is determined for the basic equivalent circuit to approximate the behavior of the negative-resistance device. The stability regions which come from the homogeneous linear second-order differential equation pertain to the operation of the negative-resistance device. When the negative-resistance device is operated as an oscillator, the instantaneous operating point describes a limit cycle for steady state conditions. The theory of the current-controlled negative-resistance device is explained using the glow discharge tube as an example. The theory of the voltage-controlled negative-resistance device is partially explained and partially simulated when the vacuum tube tetrode is used for an example.

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## SECTION I

### INTRODUCTION

This report discusses some aspects of the dynamic behavior of an open-circuit stable, two-terminal negative-resistance device with its particular equivalent circuit as well as the dynamic behavior of short-circuit stable, two-terminal negative-resistance device with its particular equivalent circuit. The equivalent circuits for these two devices are duals of each other. The static current/voltage characteristic curves for the above negative-resistance devices influence their dynamic equivalent circuits.

The currents and voltages that describe the behavior of the system are dependent upon the values of the parameters such as resistance, inductance, and capacitance, as well as time. The operation of the system is described mathematically by writing equations where the currents and voltages appear either directly or as derivatives or integrals with respect to time. The constants in the equations are determined by the parameters of the system. If a single dependent variable is sufficient to describe the behavior, only derivatives and integrals involving this one quantity appear. It is often possible to eliminate any integrals by differentiation. The resulting equations having only derivatives with respect to one variable such as time are called ordinary differential equations.

If the dependent variables such as current and voltage or their derivatives appear only to the first power, the differential equations are linear. If powers other than the first appear, or if the variables appear as products with one another or with their derivatives, the differential equations are nonlinear. The equation describing approximately a circuit with a current-controlled negative-resistance device as shown in Figure 9-1 is a nonlinear differential equation

$$\frac{d^2 i}{dt^2} + \left[ \frac{1}{R_c C_c} - \frac{a}{L_c} + \frac{3bi^2}{L_c} \right] \frac{di}{dt} + \frac{1}{L_c C_c} \left[ 1 - \frac{a}{R_c} + \frac{bi^2}{R_c} \right] i(t) = 0$$

where the voltage across the terminals of the device is equal to the function of the current through the terminals.

$$v = -a + bi^3$$

The terms involving  $\frac{3bi^2}{L_c} \frac{di}{dt}$  and  $\frac{bi^3}{L_c C_c R_c}$  are nonlinear terms.

The solution for a differential equation is a direct function of the dependent variable. The most common type of differential equation where solutions are easily found is a linear differential equation with constant coefficients.

In the case of a negative-resistance device, one of the first-order differential equations is so complicated that an exact solution is not possible. It was necessary to simplify the equation by the use of Taylor's Series. Higher than first-order derivatives were neglected by assuming that the variations of signal are small. Consequently, the slope of current/voltage characteristic curve is assumed to be approximately a constant. This assumption leads to a simplified equation. While this equation did not describe the situation exactly, it described important features of the phenomenon of the device. The simplified equation is combined with a linear first-order differential equation to obtain a second-order homogeneous linear differential equation which describes the behavior of the negative-resistance device.

This report consists of relating the dynamic and static characteristics of some negative-resistance devices by deriving expressions for their input impedances and admittances as well as by deriving the second-order differential equation based on the physical properties of the device.

This report is intended to provide the engineer and scientist with a better understanding of negative-resistance devices. The operation of the negative-resistance device for small varying signals can at least to the first order be described by piecewise linear differential equations with constant coefficients. These equations can be solved by applying certain relatively simple rules. Comparatively speaking, this report is a very small contribution when measured against the size of the field which it touches. The only purpose has been to present a readable and easily understandable report which relates the static current/voltage characteristic curve for the negative-resistance device to the dynamic equivalent circuit for the device.

The general organization of this report is along the following lines: Section II discusses some of the differences between positive-resistance devices and negative-resistance devices. Section III explains the physical reasons for the static current/voltage characteristic curve of the negative-resistance device to have a limited negative-slope region and shows the difference between the characteristic curves for the current-controlled and voltage-controlled negative-resistance devices. Section IV derives the equivalent circuit for the two types of negative-resistance devices by knowing certain information about the device. Section V explains the principle of duality. Section VI derives the basic equivalent circuit for the negative-resistance device by assuming a cause - effect relationship between the current and the voltage of the device. Section VII states a few of the properties for stability of negative-resistance devices. Section VIII explains the operation of a negative-resistance device which is used as a trigger circuit and shows the relationship between the static characteristic curve and the basic dynamic equivalent circuit. Section IX derives the linear, second order, differential, homogeneous equation which describes the limited behavior of the negative-resistance device. Section X examines which type of reactive element is essential in the basic

equivalent circuit to approximate the behavior of the negative-resistance device. Section XI discusses the different stability regions for the operation of the negative-resistance device. Section XII explains the behavior of a negative-resistance device which is used as an oscillator by showing the instantaneous operating point describing a limit cycle for steady state conditions. Section XIII explains the theory for the current-controlled negative-resistance device by using the glow discharge tube. Section XIV explains the theory for the voltage-controlled negative-resistance device by using the vacuum tube tetrode. Section XV is a list of some of the different types of negative-resistance devices.

## SECTION II

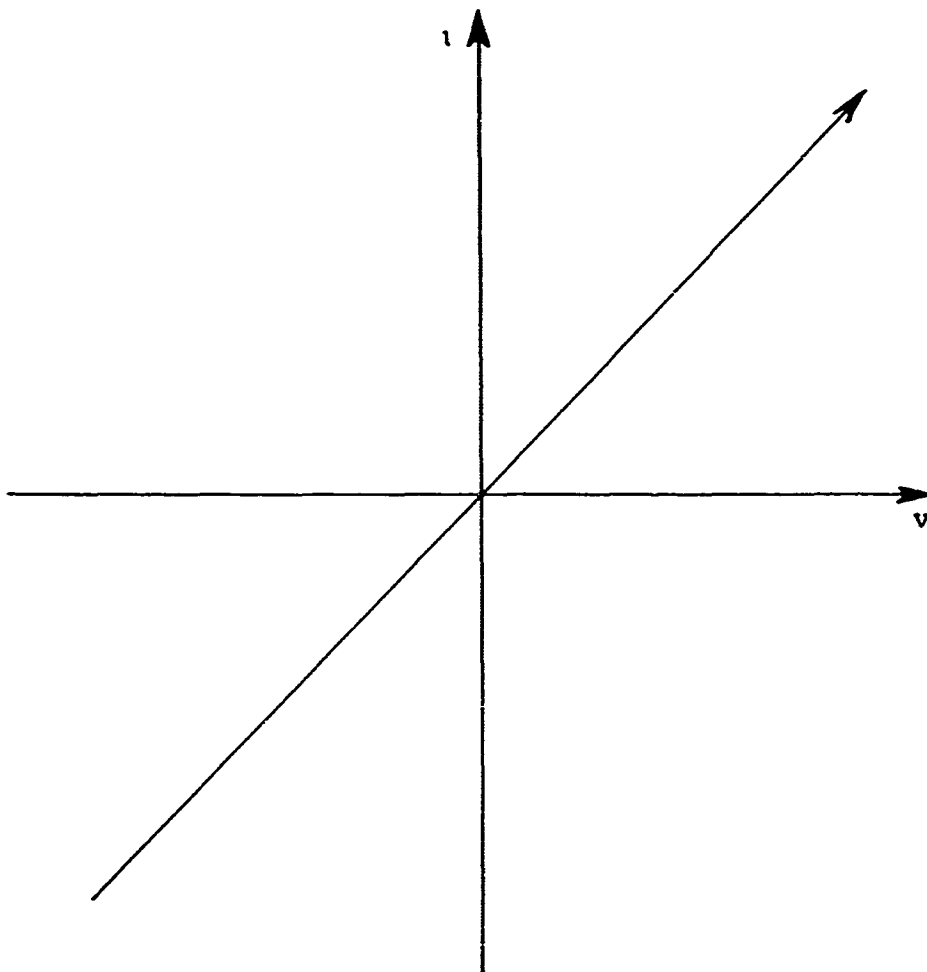
### NEGATIVE AND POSITIVE RESISTANCE

Negative resistance indicates that the slope of the current/voltage characteristic curve has a negative sign, whereas positive resistance indicates that the slope of the characteristic curve has a positive sign. The slope of the characteristic curve for a positive-resistance device is positive for the entire range of operation, as shown in Figure 2-1. Devices which possess a negative resistance are limited in the energy that they can handle, since these devices exhibit a negative resistance only over a restricted range. This is a property that is not associated with positive-resistance devices. Both of these devices are dependent on quantities, such as temperature, type of material, etc. The slope of the characteristic curve for a metallic resistor decreases with an increase in temperature for a certain range. The magnitude of the slope in the negative-resistance portion of the characteristic curve for a germanium tunnel diode increases with an increase in temperature for a particular range. A positive resistance dissipates energy; however, a negative resistance generates energy proportional to the square of the a.c. current through or the a.c. voltage across the terminals of the device. A negative resistance must have a source of energy and a means of controlling it in order to generate energy.

A physically realizable negative-resistance device is not the reverse of a positive-resistance device which has an almost linear interdependent current/voltage relationship as shown in Figure 2-1. A negative-resistance device utilizes an inherent phenomenon which does not have a linear interdependent current/voltage relationship. A current-controlled negative-resistance device operating in the negative-slope portion of its characteristic curve is a single-valued function of current and a triple-valued function of voltage. A voltage-controlled negative-resistance device operating in the negative-slope region of its characteristic is a single-valued function of voltage and a triple-valued function of current. The current/voltage characteristic curves for the two types of devices are shown in Figure 3-1.

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A good explanation on negative and positive resistance is given by Herold (14).



**Figure 2-1. Linear Current/Voltage Characteristic Curve for a Positive-Resistance Device**

## SECTION III

### CHARACTERISTIC CURVE

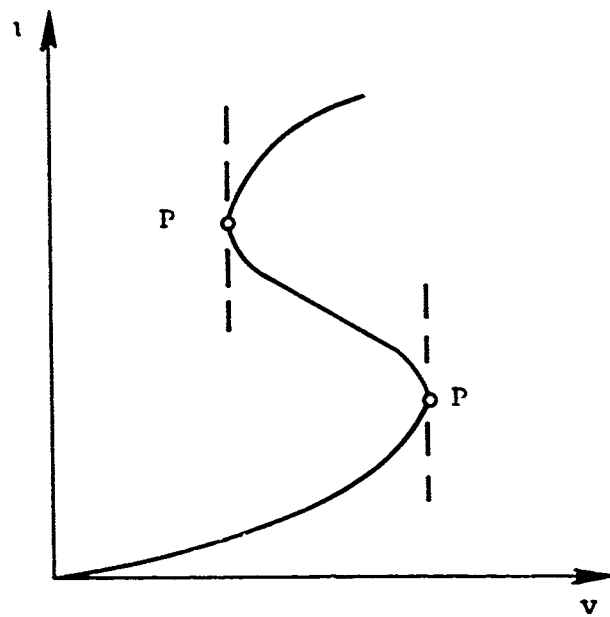
An ideal negative-resistance device is completely amplitude and frequency independent, however a real negative-resistance device has a resistive cut-off frequency above which it cannot amplify any longer. A real device must have a current/voltage characteristic curve with positive slope for high current and voltage as shown in Figure 3-1, because it could not handle infinite power. The region of interest has a negative slope which is equal to the reciprocal of the incremental negative resistance of the device. The characteristic curve cannot have a negative slope at the origin. If there were a negative slope at the origin, the result would be power output without any power input. This would violate the principle of the conservation of energy. Consequently, the region of negative slope must be inserted between the regions of positive slope. The transition between the positive-slope regions and the negative-slope region can be made only in the two ways shown in Figure 3-1. At a point P between regions of positive and negative slope, the characteristic curve for the current-controlled negative-resistance device has an infinite slope which is equal to zero incremental resistance. Similarly, the characteristic curve for the voltage-controlled negative-resistance device has a zero slope which is equal to infinite incremental resistance.

The static current/voltage characteristic curve shows that the current-controlled negative-resistance device is uniquely controlled by the current passing through the terminals of the device, since its characteristic curve is a single-valued function of current. Analogously, the voltage-controlled negative-resistance device is uniquely controlled by the voltage across the terminals of the device, because its characteristic curve is a single-valued function of voltage. Control by the other variable is impossible, since the static current/voltage characteristic curve is triple-valued in the negative-slope region. The characteristic curve for the current-controlled negative-resistance device is called an S-type curve and for the voltage-controlled negative-resistance device is called an N-type curve.

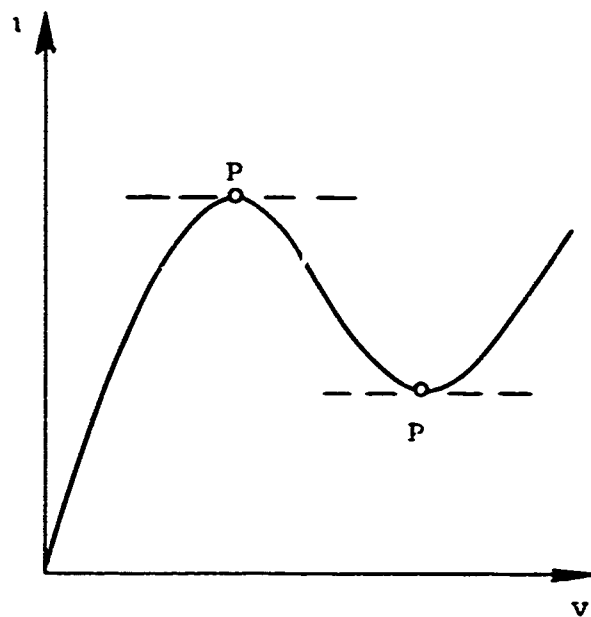
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There are many references which deal with the static current, voltage characteristic curve for negative-resistance devices. Two excellent ones are Hall (13) and Thompson (31).





Current-Controlled Negative-Resistance  
Characteristic Curve  
 $v = f(i)$



Voltage-Controlled Negative-Resistance  
Characteristic Curve  
 $i = f(v)$

Figure 3-1.

## SECTION IV

### EQUIVALENT CIRCUIT

The current-controlled and the voltage-controlled negative-resistance devices have dynamic equivalent circuits which correspond to their frequency dependent properties. The purpose of an equivalent circuit is to represent the frequency dependence of a device by means of frequency independent parameters and related formulas. For example, consider a two-terminal device which can amplify or oscillate, is open-circuit stable, and requires d. c. bias current. Since the device is open-circuit stable, it is stable with a current source at the input terminals. The amplifying mechanism of the device is represented by a current-controlled negative resistance and the remaining parameters by a four-terminal black box as shown in Figure 4-1. It is assumed that the negative resistance is frequency independent. All known two-terminal, negative-resistance devices have a resistive cut-off frequency above which they cannot amplify any longer. If the black box did not contain any loss resistance, then a finite cut-off frequency would not exist. The loss resistance cannot appear directly in series or parallel with the negative resistance. If it did, the loss resistance would be frequency independent, and the resistive cut-off frequency would still have to be infinite or zero. Consequently, there would have to be a reactance in the box adjoining the negative resistance. The d. c. bias current requirement eliminates series capacitors and shunt inductors, therefore a low pass structure is required. The next element must be either a shunt capacitor or a series inductor. If it were a shunt capacitor, then at some adequately high frequency the input impedance as shown in Figure 4-2 could be very small. Then the device would be unstable. Since it was assumed to be open-circuit stable, the shunt capacitor is excluded. Therefore, the second element of the equivalent circuit must be a series inductance as shown in Figure 4-3.

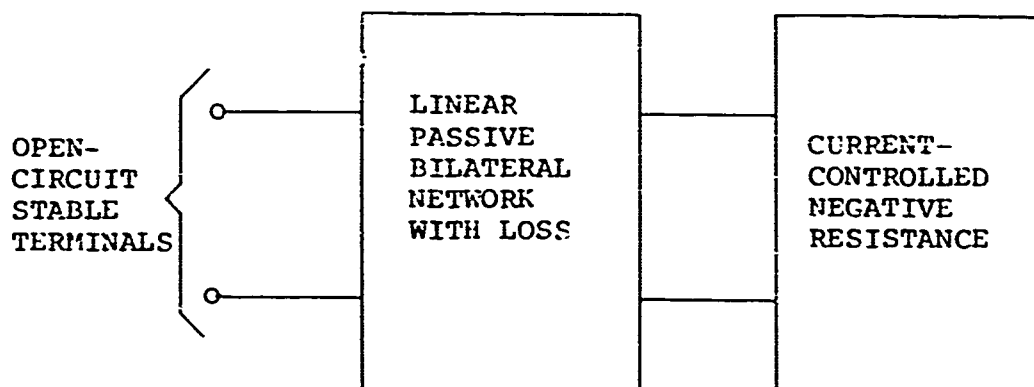


Figure 4-1. Representation of An Open-Circuit Stable Two-Terminal Device

The equivalent circuit for the current-controlled and voltage-controlled negative-resistance devices is derived by recognizing particular information about the device. There is a fine discussion by Hall (13).

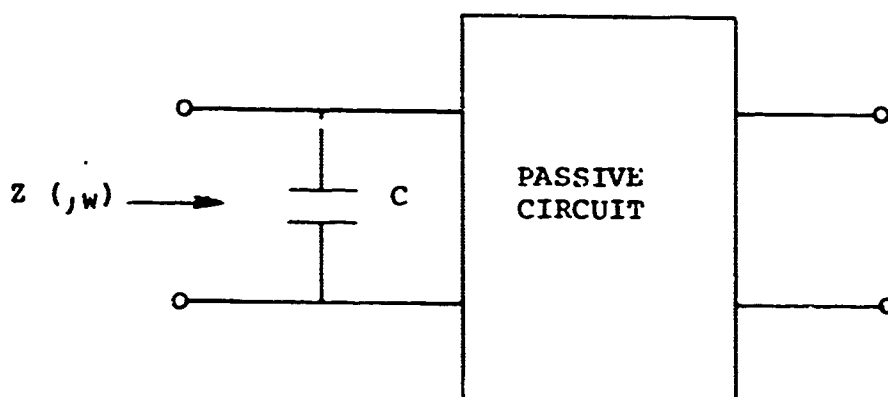


Figure 4-2. Equivalent Circuit of a Low Impedance Device at a High Frequency

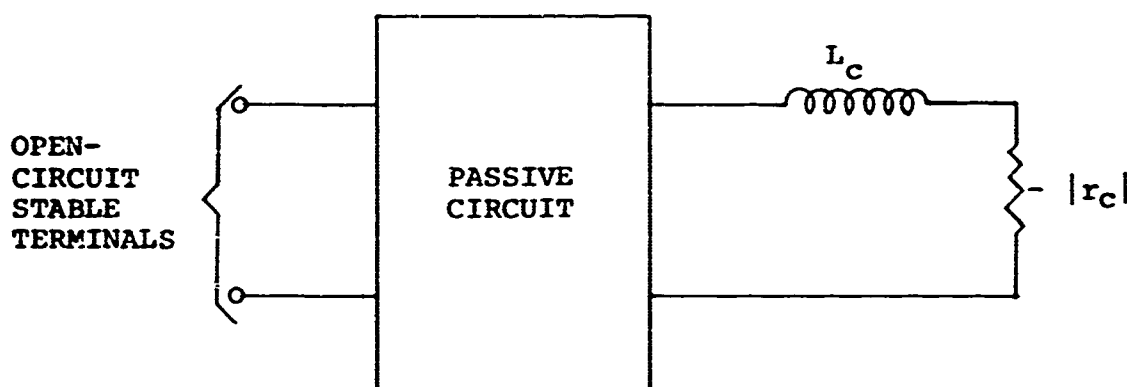


Figure 4-3. Basic Equivalent Circuit of Open-Circuit Stable Negative-Resistance Device

Positive resistance or loss can now be introduced as a shunt resistance to yield a finite resistive cut-off frequency. Shunt capacitance can be inserted in the equivalent circuit to produce a resonant frequency or self-frequency of oscillation. From here on, any low-pass structure of series inductors, shunt capacitors, and shunt resistors is possible. This equivalent circuit as shown in Figure 4-4 is typical for current-controlled negative-resistance devices.

Analogous reasoning applies to a two-terminal device which can amplify or oscillate, is short-circuit stable, and requires d. c. bias voltage. Because the device is short-circuit stable, it is stable with a voltage generator at the input terminals. The amplifying mechanism of the device is represented by a voltage-controlled negative resistance and the remaining parameters by a four-terminal black box as shown in Figure 4-5. It is assumed that the negative resistance is frequency independent. Shunt inductors and series capacitors are eliminated because of the bias voltage requirement. The next

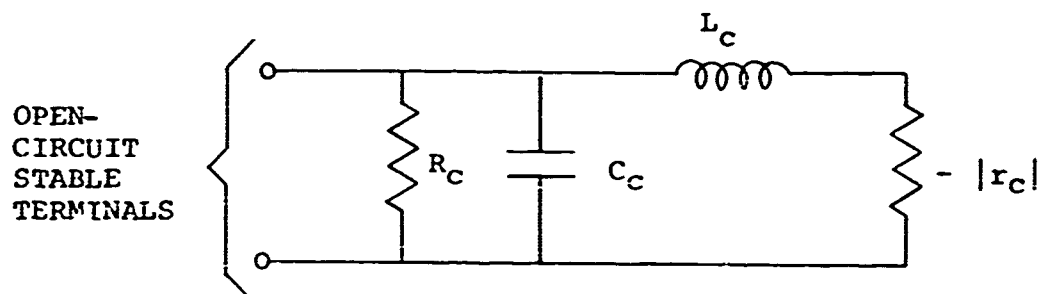


Figure 4-4. Complete Equivalent Circuit of the Open-Circuit Stable, Two-Terminal Negative-Resistance Device

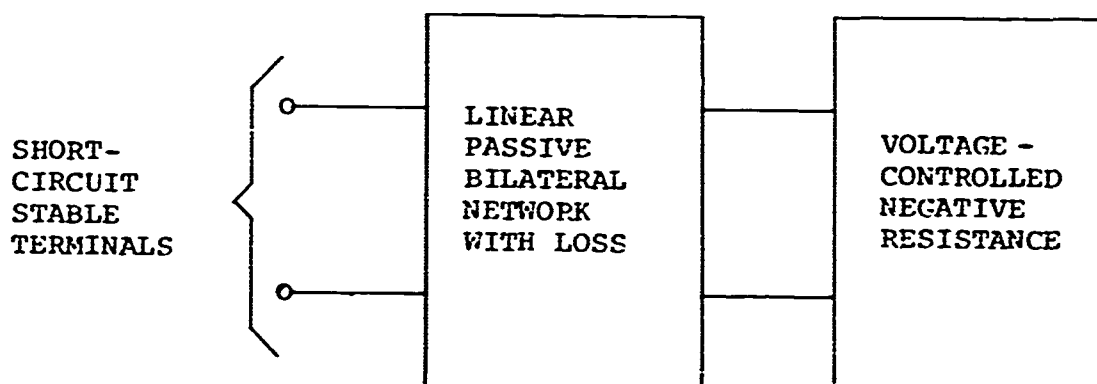


Figure 4-5. Representation of a Short-Circuit Stable, Two-Terminal Device

element must be either a series inductor or a shunt capacitor. If it were a series inductor, then at some sufficiently high frequency the input impedance as shown in Figure 4-6 could be very large. Evidently the device would be unstable. Because it was supposed to be short-circuit stable, the series inductor is eliminated. Consequently, the second element of the equivalent circuit must be a shunt capacitor as shown in Figure 4-7. Positive resistance can be introduced as a series resistance to produce a finite resistive cut-off frequency. Series inductance is inserted to yield a self-frequency of oscillation. This equivalent circuit as shown in Figure 4-8 is typical for voltage-controlled negative-resistance devices.

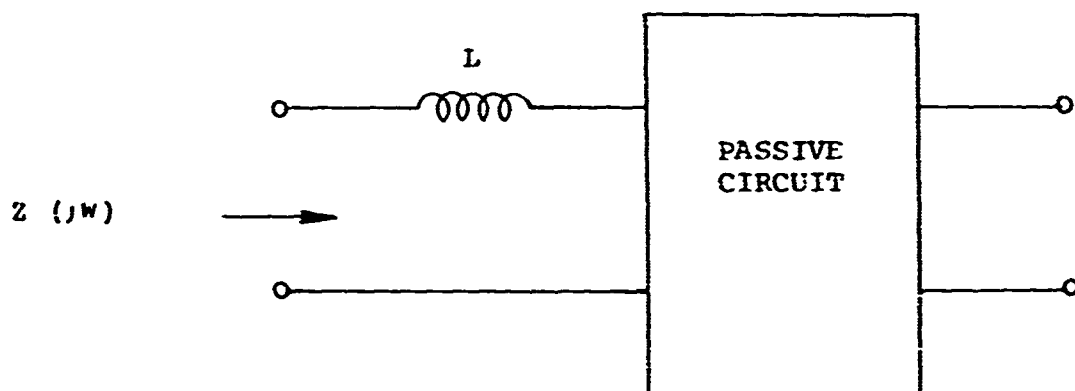


Figure 4-6. Equivalent Circuit of a High-Impedance Device at a High Frequency

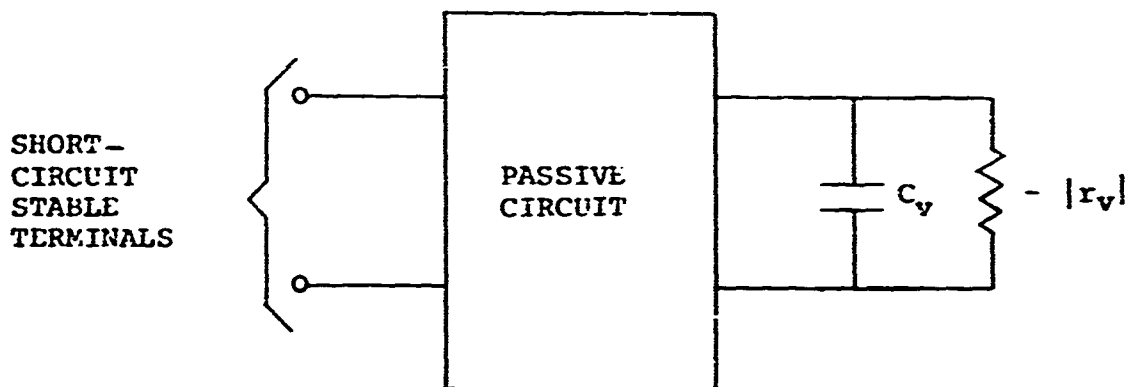


Figure 4-7. Basic Equivalent Circuit of Short-Circuit Stable Negative-Resistance Device

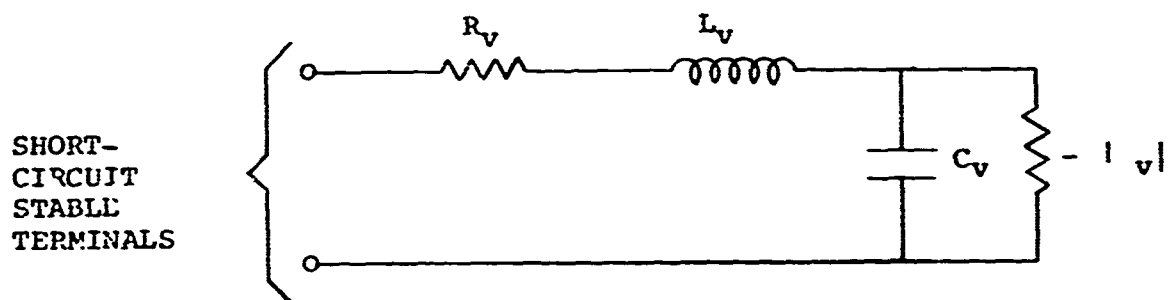


Figure 4-8. Complete Equivalent Circuit of Short-Circuit Stable, Two-Terminal Negative-Resistance Device

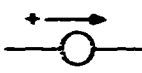

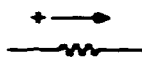
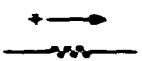
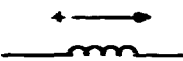
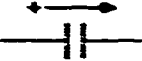
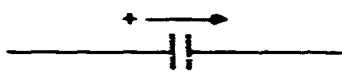
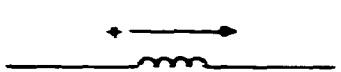
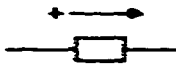
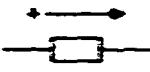
## SECTION V

### DUALITY

Two network elements are dual if they play similar roles in the formation of the laws and procedures of network theory which come under the general heading of loop analysis and node analysis.

Two branches are dual if the expression for voltage in terms of current of the first branch has the same form as the expression for the current in terms of the voltage of the second branch. The dual pairs of current-voltage relationships are shown in Table 5-1.

TABLE 5-1. DUAL BRANCHES

 $v(t) = v_g(t)$	 $i(t) = i_g(t)$
 $v(t) = i(t)R$	 $i(t) = v(t)G$
 $v(t) = L \frac{di}{dt}$	 $i(t) = C \frac{dv}{dt}$
 $v(t) = \frac{1}{C} \int_0^t i(x) dx - v(0^-)$	 $i(t) = \frac{1}{L} \int_0^t v(x) dx - i(0^-)$
 $v = \tilde{i}(\tilde{v})$	 $i = \tilde{i}(v)$

There are a great many books in the field of network theory which explain the principle of duality. Two worthy ones are Balabanian (1) as well as Sedra and Balabanian (27).

Two networks, N1 and N2 are dual networks if the following conditions are satisfied:

1. The current equations of N1 are the same as the voltage equations of N2, provided current is replaced by voltage.
2. The voltage equations of N1 are the same as the current equations of N2, provided voltage is replaced by current.
3. The branches of N1 are the duals of the corresponding branches of N2. A list of some dual quantities and laws is given in Table 5-2.

In some of the succeeding sections, the concept of duality is used to obtain results concerning one network when the corresponding results for the dual network are available.

TABLE 5-2. DUAL QUANTITIES AND LAWS

Voltage	Current
Kirchhoff's voltage law	Kirchhoff's current law
Loop	Node
Number of loops	Number of nodes
Node voltage	Loop current
Resistance	Conductance
Inductance	Capacitance
Series	Parallel
Short-Circuit	Open-Circuit
Loop equations	Node equations

## SECTION VI

### STRAY REACTIVE ELEMENTS RELATED TO TIME DELAYS

This section explains the requirement for energy-storage elements in the small-signal dynamic equivalent circuits for negative-resistance devices in terms of the time delay in the mechanism of behavior. It is assumed that the cause leads the effect by a short time interval since the behavior of the device can be approximated with an appropriate inductor or capacitor in the equivalent circuit. In a current-controlled negative-resistance device where the voltage is a single-valued function of the current, it is assumed that the cause is the current and the effect is the voltage. Similarly, in a voltage-controlled negative-resistance device where the current is a unique function of the voltage, it is assumed that the cause is the voltage and the effect is the current.

If a negative-resistance device has a static current/voltage characteristic curve that is continuous and possesses a continuous first derivative, this characteristic curve can be approximated by any small segment which has the following current-voltage relationships

$$I = a + bV \quad (6-1)$$

or

$$V = d + hI \quad (6-2)$$

The constants  $d$  and  $h$  can be expressed in terms of the constants  $a$  and  $b$ , or vice versa. In the negative slope region  $b$  and  $h$  are negative.

When the current and voltage become functions of time, equations 6-1 and 6-2 can be written as

$$i(t) = a + bv(t - t_0) \quad (6-3)$$

or

$$v(t) = d + hi(t - t_0) \quad (6-4)$$

depending on whether the current or voltage is the cause and the voltage or current is the effect. Equations 6-1 and 6-2 are sound for both types of characteristic curves,

---

The derivation of the basic equivalent circuit for the current-controlled and voltage-controlled negative-resistance devices from the Taylor Series is discussed by Card (9).



but equation 6-3 is valid only for an N-type characteristic curve in the region where its negative slope is linear. Analogously, equation 6-4 is valid only for an S-type characteristic curve in the region where its negative slope is linear. Implied in the above statements are the assumptions that current-controlled and single-valued function of current are equivalent, as well as voltage-controlled and single-valued function of voltage are equivalent. For example, the voltage across the terminals of a glow discharge tube in the negative slope-region at a time  $t$  should depend on the current through the terminals of the tube  $t_0$  seconds earlier. The current through the terminals of a vacuum tube tetrode in the negative-slope region at a time  $t$  should depend on the voltage across the terminals of the tube  $t_0$  seconds earlier.

Once it has been determined to use the appropriate one of the equations 6-3 or 6-4, then the type of energy storage element comes from a straight-forward derivation. Consider the current-controlled negative-resistance device and expand  $i(t - t_0)$  in a Taylor Series about  $t$ :

$$i(t - t_0) = i(t) - t_0 \frac{di}{dt} + \frac{t_0^2}{2!} \frac{d^2i}{dt^2} + \dots$$

If the terms involving higher than first-order derivatives are neglected, equation 6-4 can be written as

$$v(t) = d + h i(t) - h t_0 \frac{di}{dt}$$

$$\text{let } d = V_0, h = r_c, \text{ and } -h t_0 = L_c$$

$$v(t) = V_0 + r_c i(t) + L_c \frac{di}{dt} \quad (6-5)$$

The inductor  $L_c$  is positive because  $h$  is negative in the negative-slope region. The basic small-signal dynamic-equivalent circuit for the current-controlled negative-resistance device as shown in Figure 4-3 is implied in equation 6-5.

Similarly, consider the voltage-controlled negative-resistance device and expand  $v(t - t_0)$  in a Taylor Series about  $t$ :

$$v(t - t_0) = v(t) - t_0 \frac{dv}{dt} + \frac{t_0^2}{2!} \frac{d^2v}{dt^2} + \dots$$

If the terms involving higher than first order derivatives are neglected, equation 6-3 can be written as

$$i(t) = a + bv(t) - bt_o \frac{dv}{dt}$$

$$\text{let } a = I_o, \quad b = \frac{1}{r_v}, \quad \text{and } -bt_o = C_v$$

$$i(t) = I_o + \frac{v(t)}{r_v} + C_v \frac{dv}{dt} \quad (6-6)$$

The capacitor  $C_v$  is positive because  $b$  is negative in the negative-slope region. The basic small-signal dynamic-equivalent circuit for the voltage-controlled negative-resistance device as shown in Figure 4-7 is implied in equation 6-6.

Equations 6-5 and 6-6 are valid only in the negative-slope region because it is preferred that an equivalent circuit has positive values for the energy storage elements.

It is emphasized that the source of the observed energy storage elements related to the negative-resistance devices must be partially in the mechanism of behavior rather than exclusively in recognizable geometric properties. Therefore, the observed inductance of a glow discharge tube biased in the negative-slope region does not indicate energy stored in a magnetic field. The observed capacitance of a vacuum tube tetrode biased in the negative-slope region does not imply energy in an electric field.

## SECTION VII

### PROPERTIES OF NEGATIVE-RESISTANCE DEVICES

The following assumed properties are sufficient for stability of one-port negative-resistance devices:

- (1) The static current/voltage characteristic curve for the device exists and is continuous at every point on its characteristic curve in the region of interest. For example, if the function  $v = f(i)$  for the current-controlled negative-resistance device has a finite derivative

$$f'(i_1) = \frac{df(i_1)}{di} = \lim_{\Delta i \rightarrow 0} \frac{f(i_1 + \Delta i) - f(i_1)}{\Delta i}$$

at  $i = i_1$  then  $f(i)$  is continuous at  $i = i_1$

- (2) For any operating point on the current/voltage characteristic curve, the ratio of small-signal complex variational currents and voltages is a rational function of  $S$  with real coefficients. This impedance or admittance function is not positive real in the negative-slope region of the characteristic curve.

The following conditions are necessary and sufficient for the positive realness of a rational function of the complex variable  $S$ :

- (a) The function is regular in the right half  $S$ -plane;
- (b) Any poles on the  $j\omega$ -axis are simple with real positive residues;
- (c) The real part of the function is non-negative on the  $j\omega$ -axis.

The reciprocal of a positive real function is also positive real. For example, the admittance function

$$Y_c(S) = C_c \left[ \frac{S^2 + S(1/R_c C_c - r_c/L_c) + 1/L_c C_c (1 + r_c/R_c)}{S + r_c/L_c} \right]$$

The properties for the stability of negative-resistance devices are stated by Card (9). The conditions for positive realness of a rational function are considered by Balabanian (2).

for the small-signal equivalent circuit for the current-controlled negative-resistance device and the impedance function

$$Z_V(S) = L_V \left[ \frac{S^2 + S(R_V/L_V + 1/r_V C_V) + 1/L_V C_V (1 + R_V/r_V)}{S + 1/r_V C_V} \right]$$

for the small-signal equivalent circuit for the voltage-controlled negative-resistance device are rational functions of  $S$  with real coefficients. The real part of the admittance of the current-controlled device

$$\operatorname{Re} [Y_C(j\omega)] = \frac{\omega^2 + r_C^2/L_C^2 + R_C r_C/L_C^2}{R_C(\omega^2 + r_C^2/L_C^2)}$$

and the real part of the impedance of the voltage-controlled device

$$\operatorname{Re} [Z_V(j\omega)] = \frac{R_V(\omega^2 + 1/r_V^2 C_V^2 + 1/R_V r_V C_V^2)}{\omega^2 + 1/r_V^2 C_V^2}$$

may be negative on the  $j\omega$ -axis in the negative-slope region. Therefore, the admittance function for the current-controlled device and the impedance function for the voltage-controlled device are not positive real in the negative-slope region of the characteristic curve.

- (3) At an adequately high frequency the real part of the impedance or admittance function becomes positive. For example, when the frequency approaches infinity, the real part of the admittance function for the current-controlled device is equal to the reciprocal of the shunt loss resistance

$$\lim_{\omega \rightarrow \infty} \operatorname{Re} [Y_C(j\omega)] = \frac{1}{R_C}$$

and the real part of the impedance function for the voltage-controlled device is equal to the series loss resistance

$$\lim_{\omega \rightarrow \infty} \operatorname{Re} [Z_V(j\omega)] = R_V$$

- (4) The degrees of the numerator and denominator do not differ by more than one for the impedance or admittance functions. For example, the admittance function for the current-controlled device and the impedance for the voltage-controlled device have a numerator that is a second-degree polynomial as well as a denominator that is a first-degree polynomial. Consequently, the degrees of the numerator and denominator of these functions differ by one.

- (5) The negative-resistance device is stable in the negative-slope region of the characteristic curve with either a current source or a voltage source. The one port network which has its admittance connected to a current generator is open-circuit stable if the admittance function of the network does not possess any zeros in the right half S-plane. Analogously, the one-port network which has its impedance connected to a voltage generator is short-circuit stable, if the impedance function of the network does not possess any zeros in the right half S-plane. For example, in the negative-slope region, the admittance of the equivalent circuit for the current-controlled negative-resistance device  $Y_C(S)$  is open-circuit stable. Similarly, in the negative-slope region, the impedance of the equivalent circuit for the voltage-controlled negative-resistance device  $Z_V(S)$  is short-circuit stable.

Small-signal stability is related to the internal resistance of the source. The operation of a current-controlled negative-resistance device is stable in the negative-slope region of its characteristic curve, if this device is driven by a generator with high internal resistance. The operation of a voltage-controlled negative-resistance device is stable in the negative-slope region if this device is driven by a source with low internal resistance. It is assumed in the above two statements that other conditions for stable operation are satisfied.

## SECTION VIII

### TRIGGER CIRCUIT

A negative-resistance device may be used for certain types of trigger circuits which are characterized by having two stable states separated by an unstable state. The circuit can be made to switch from one stable state to the other by applying the suitable trigger pulse. During the switching process which has a definite time interval, the circuit passes through an unstable condition.

If a current-controlled negative-resistance device is driven from a generator that has a low internal resistance, the device can be used as a trigger circuit. For a value of bias current  $I$  and a load resistance  $R_L$ , there are three equilibrium points as shown in Figure 8-1. The two intersections on the positive-slope portions of the S-type curve are stable and the other intersection in the negative-slope region is unstable.

It will be assumed that initially the current-controlled negative-resistance device is at a stable equilibrium point on the lower positive-slope region as shown in Figure 8-1. If the bias current is raised momentarily by a positive trigger pulse, the load line is shifted upward and to the right by a certain amount as shown in Figure 8-2. The operating point begins to move along the new load line toward the stable equilibrium point on the upper positive-slope region. It is assumed that the trigger pulse is maintained long enough so that the operating point current exceeds the current value of the unstable equilibrium point. At the end of the trigger pulse, the bias current is lowered to its original value which causes the load line to shift to its original position on the current/voltage characteristic curve. If the bias current is lowered momentarily by a negative trigger pulse, the operating point will switch back to the stable equilibrium point on the lower positive-slope region.

It is usually desirable that the time required for the shift from one stable state to the other be as small as possible. This time is dependent upon the magnitudes of the energy storage elements that are associated with negative-resistance devices. If these elements are reduced in size, a smaller transition time will result between stable states.

When a current-controlled negative-resistance device is connected to a source with a very low internal resistance, it is observed by measurements and on the characteristic curve as shown in Figure 8-3 that the current through the terminals of the device cannot change instantaneously during the switching process. Consequently, the basic

A first-class explanation of the voltage-controlled negative-resistance device which is operated as a trigger circuit is found in Cunningham (12) as well as Nanavati, Card, Harbourt and Glassford (24).

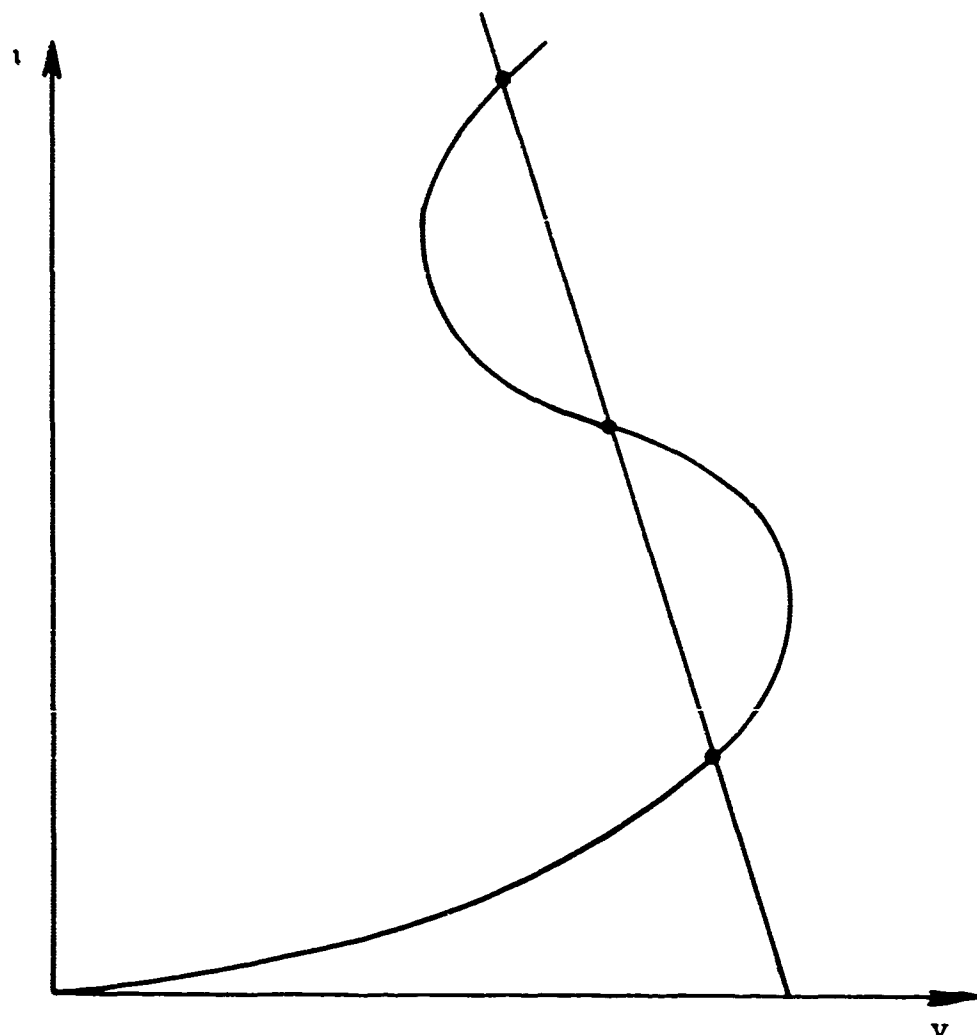


Figure 8-1. Typical S-Curve for a Current-Controlled Negative-Resistance Device with Three Intersections by a Load Line

equivalent circuit for the current-controlled negative-resistance device has an inductor in series with the negative resistance as shown in Figure 4-3. The d.c. bias current requirement eliminated shunt inductors.

Similarly, when a voltage-controlled negative-resistance device is connected to a generator with a very high internal resistance, it is observed by measurements and on the characteristic curve as shown in Figure 8-4 that the voltage across the terminals of the device cannot change instantaneously during the switching process. Therefore, the basic equivalent circuit for the voltage-controlled negative-resistance device has a capacitor in parallel with the negative resistance as shown in Figure 4-7. The d.c. bias voltage requirement eliminated series capacitors.

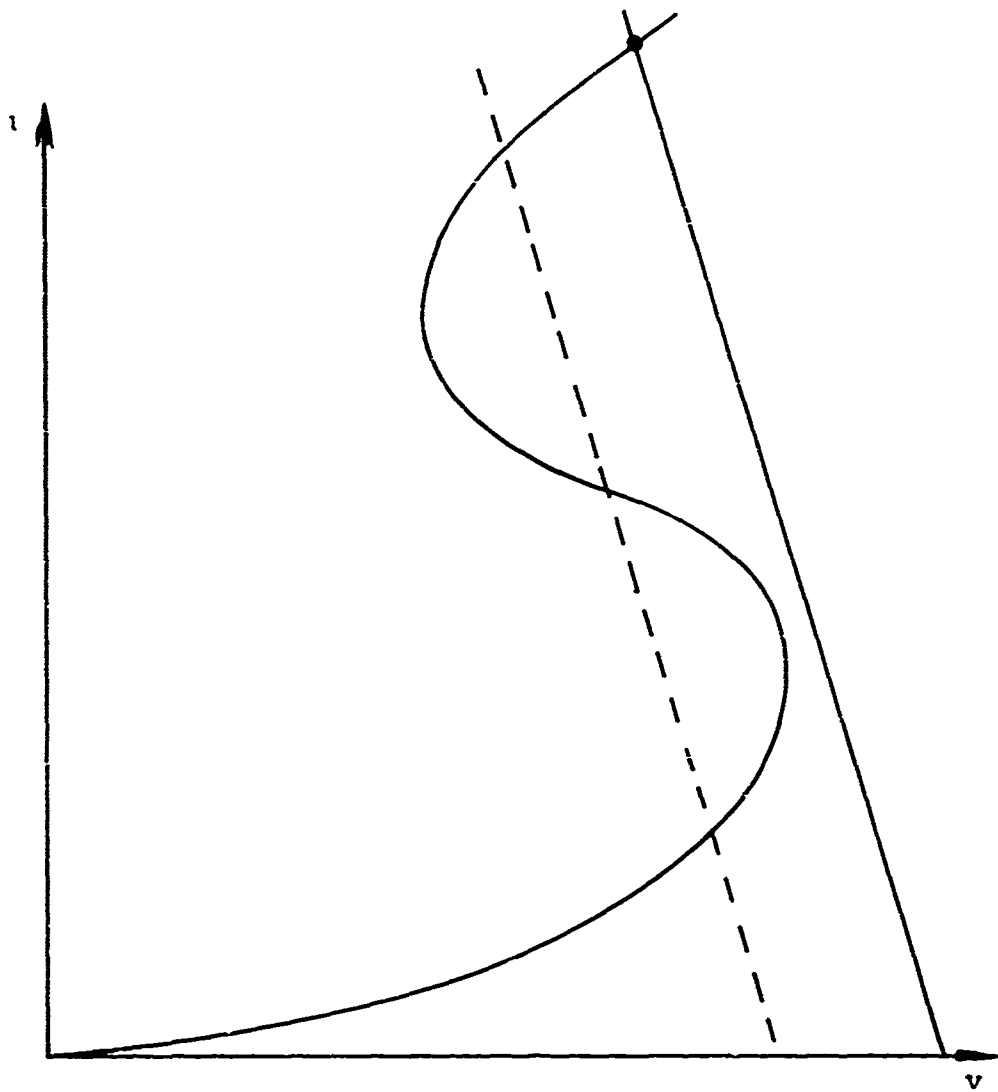


Figure 8-2. S-Curve with Load Line Momentarily Shifted Upward and to the Right



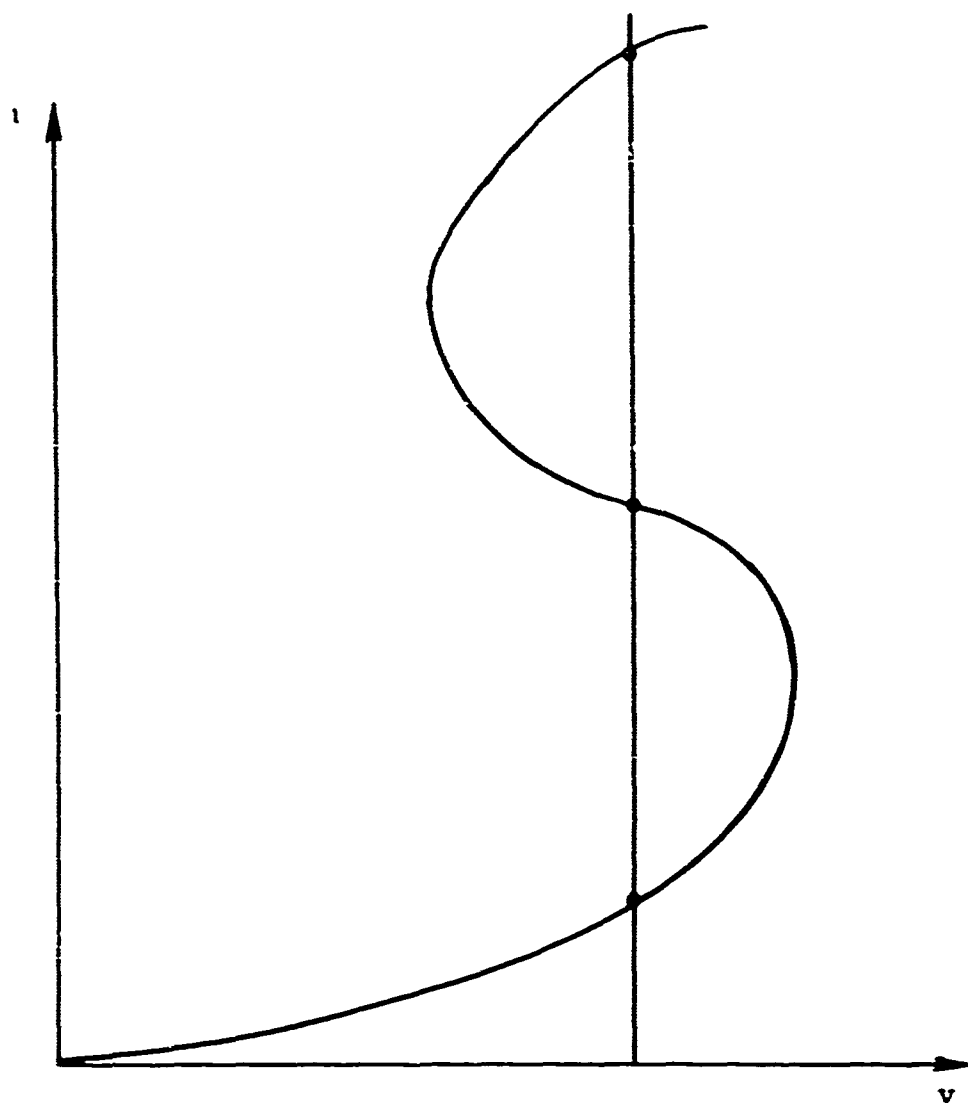


Figure 8-3. S-Type Curve with Three Intersections by a Load Line of Infinite Slope

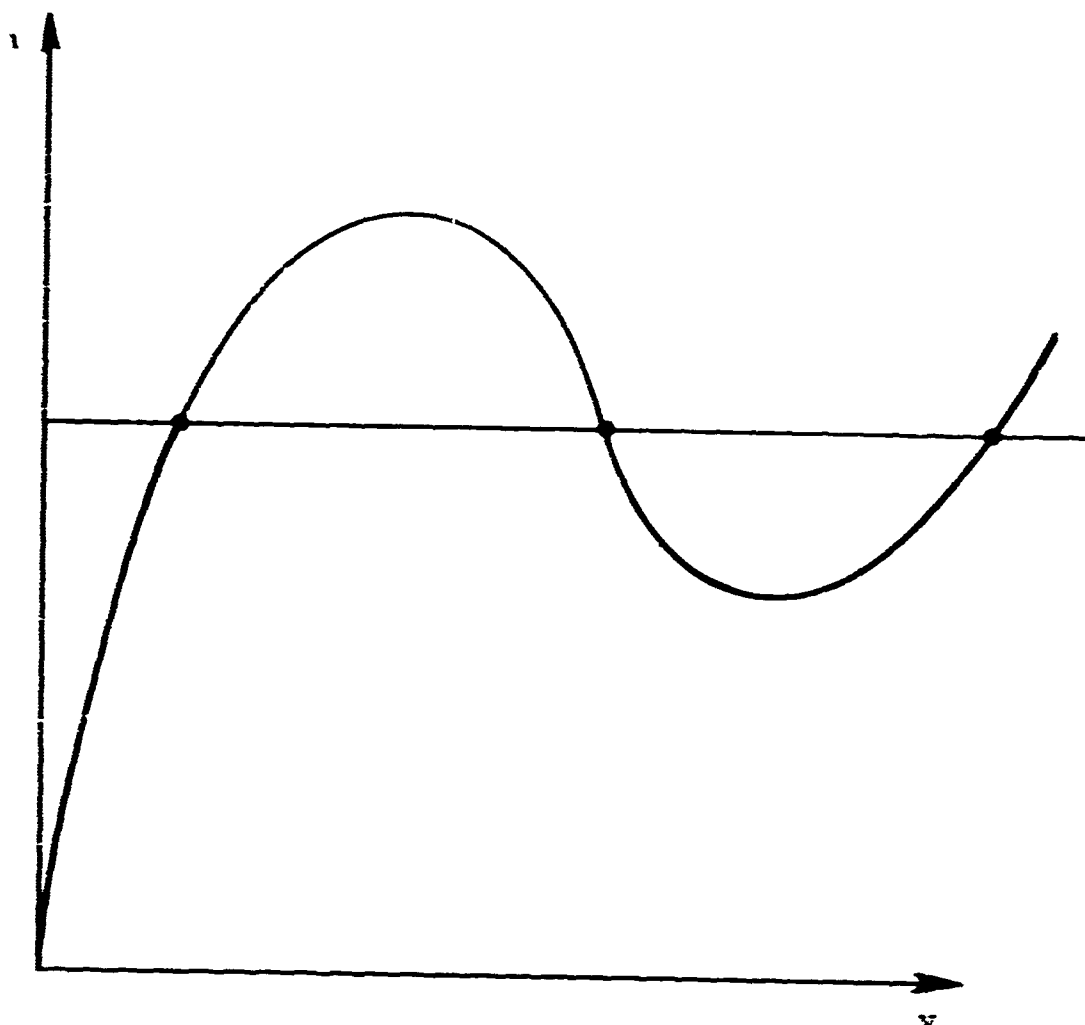


Figure 8-4. N-Type Curve with Three Intersections by a Load Line of Zero Slope

## SECTION IX

### OSCILLATOR CIRCUIT

A simple self-excited oscillator circuit can be made by using a negative-resistance device which is operated in the negative-slope region of its current/voltage characteristic curve. The circuit can be made to oscillate, if initially the increase of small-signal power due to the dynamic negative resistance of the device is greater than the small signal power loss due to damping. Consequently, the initial a. c. signal will be amplified more than it is attenuated and will build up as an oscillation. Even if the oscillations grow initially, there can not be any growth in steady state. For steady state oscillations, the input impedance or input admittance of the dynamic equivalent circuits for the two types of negative-resistance devices is equal to zero. Therefore, the steady state a. c. signal is neither amplified nor attenuated. The equations which describe the behavior of the self-excited oscillator circuit are combined into a homogeneous linear second-order differential equation.

If a current-controlled negative-resistance device is driven from a source that has a high internal resistance and the values of the energy storage elements in the circuit are suitable, the device can be used as an oscillator circuit as shown in Figure 9-1. For the values of bias current  $I$  and load resistance  $R_c$ , there is only one equilibrium point as shown in Figure 9-3. By the proper choice of variables, steady state operation of this oscillator circuit can be described in terms of two simultaneous first-order differential equations

$$\frac{di_c}{dt} = \frac{v'(t) - f(i_c)}{L_c} \quad \text{and} \quad \frac{dv'}{dt} = \frac{I}{C_c} - \frac{v'(t)}{R_c C_c} - \frac{i_c(t)}{C_c}$$

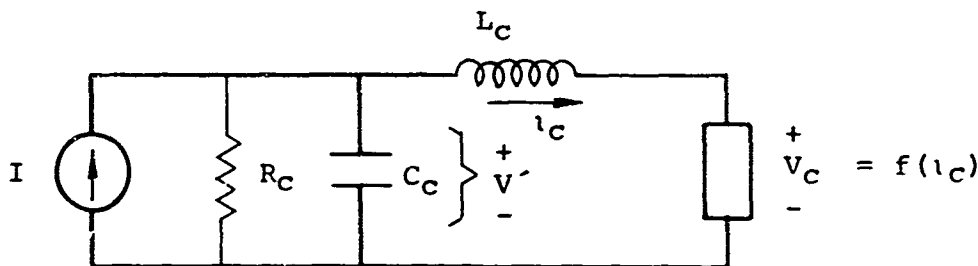


Figure 9-1. Equivalent Circuit for a Current-Controlled Negative-Resistance Device Used as an Oscillator

The linear, second order, homogeneous differential equation that describes the behavior of the negative-resistance device is derived by Cunningham (12), Minorsky (22) and Sterzer (29). The steady state oscillation frequency and the resistive cut off frequency are examined by Blaesser (7) and Nanavati (25).

The operating point exists where simultaneously

$$I = \frac{V'_s}{R_c} + I_{cs} \quad \text{and} \quad V'_s = V_{cs} = f(I_{cs})$$

The second of these conditions indicates that at the operating point the d. c. voltage across the inductor is zero. Near the operating point, the following substitutions can be made

$$i_c(t) = I_{cs} + i(t)$$

$$v'(t) = V'_s + v(t)$$

where  $i(t)$  and  $v(t)$  are small signal variations. The two simultaneous first-order equations can be written

$$\frac{dv}{dt} = -\frac{1}{C_c} \left[ i(t) + \frac{v(t)}{R_c} \right] \quad (9-1)$$

$$\frac{di}{dt} = \frac{1}{L_c} \left[ v'(t) - f(i_c) \right] \quad (9-2)$$

In equation 9-2,  $f(i_c)$  is expanded in a Taylor Series about  $I_{cs}$ :

$$f(i_c) = f(I_{cs}) + \frac{df(I_{cs})}{di_c} \left[ i_c - I_{cs} \right] + \frac{d^2 f(I_{cs})}{di_c^2} \left[ \frac{i_c - I_{cs}}{2!} \right]^2 +$$

$$\frac{d^3 f(I_{cs})}{di_c^3} \left[ \frac{i_c - I_{cs}}{3!} \right]^3 + \dots$$

The terms involving higher than first-order derivatives are neglected by assuming that the variations of the signal are small.

$$f(i_c) = f(I_{cs}) + \frac{df(I_{cs})}{di_c} \left[ i \right]$$

$$\frac{df(I_{cs})}{di_c} = r_c$$

where  $r_c$  is the incremental resistance of the device evaluated at the operating point. Equation 9-2 can be written

$$\frac{di}{dt} = \frac{1}{L_c} \left[ v(t) - i(t) r_c \right] \quad (9-3)$$

The following homogeneous, linear, second-order differential equation was obtained from equations 9-1 and 9-3.

$$\frac{d^2 i}{dt^2} + \left[ \frac{1}{R_c C_c} + \frac{r_c}{L_c} \right] \frac{di}{dt} + \frac{1}{L_c C_c} \left[ 1 + \frac{r_c}{R_c} \right] i(t) = 0 \quad (9-4)$$

This is the linear differential homogeneous equation for a current-controlled negative-resistance device.

Similarly, if a voltage-controlled negative-resistance device is driven from a generator that has a low internal resistance, the device can be used as an oscillator circuit as shown in Figure 9-2. For the values of bias voltage  $E$  and the load resistance  $R_v$ , there is one equilibrium point on the N-type curve as shown in Figure 9-4. By the principle of duality, operation of this oscillator circuit can be described in terms of two simultaneous linear first-order differential equations

$$\frac{di}{dt} = - \frac{1}{L_v} \left[ v(t) + i(t) R_v \right] \quad (9-5)$$

$$\frac{dv}{dt} = \frac{1}{C_v} \left[ i(t) - \frac{v(t)}{r_v} \right] \quad (9-6)$$

The following homogeneous linear second-order differential equation was obtained from equations 9-5 and 9-6.

$$\frac{d^2 v}{dt^2} + \left[ \frac{R_v}{L_v} + \frac{1}{r_v C_v} \right] \frac{dv}{dt} + \frac{1}{L_v C_v} \left[ 1 + \frac{R_v}{r_v} \right] v(t) = 0 \quad (9-7)$$

This is the linear differential homogeneous equation for a voltage-controlled negative-resistance device.

TABLE 9-1. CHARACTERISTIC FREQUENCIES FOR  
NEGATIVE-RESISTANCE DEVICES

The steady state oscillation frequency is the resonant frequency of the equivalent circuit for the negative-resistance device where the imaginary part of the input impedance or admittance becomes zero.

For the current-controlled negative-resistance device

$$\text{Im} \left[ Y_c(j\omega) \right] = 0$$

$$\omega_{xc} = 2\pi f_{xc} = \sqrt{1/L_c C_c - r_c^2/L_c^2}$$

For the voltage-controlled negative-resistance device

$$\text{Im} \left[ Z_v(j\omega) \right] = 0$$

$$\omega_{xc} = 2\pi f_{xc} = \sqrt{1/L_v C_v - 1/r_v^2 C_v^2}$$

The resistive cut-off frequency is the frequency which separates the active and passive operation of the equivalent circuit of the negative-resistance device where the real part of the input impedance or admittance becomes zero.

For the current-controlled negative-resistance device

$$\text{Re} \left[ Y_c(j\omega) \right] = 0$$

$$\omega_{rc} = 2\pi f_{rc} = |r_c|/L_c \sqrt{R_c/|r_c| - 1}$$

For the voltage-controlled negative-resistance device

$$\text{Re} \left[ Z_v(j\omega) \right] = 0$$

$$\omega_{rv} = 2\pi f_{rv} = 1/|r_v|C_v \sqrt{|r_v|/R_v - 1}$$

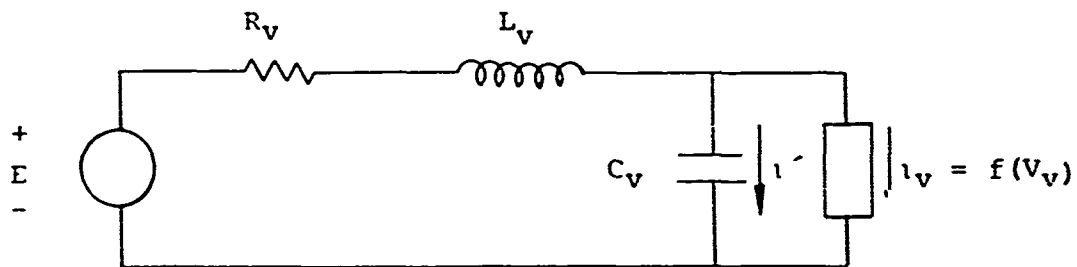


Figure 9-2. Equivalent Circuit for a Voltage-Controlled Negative-Resistance Device Used as an Oscillator

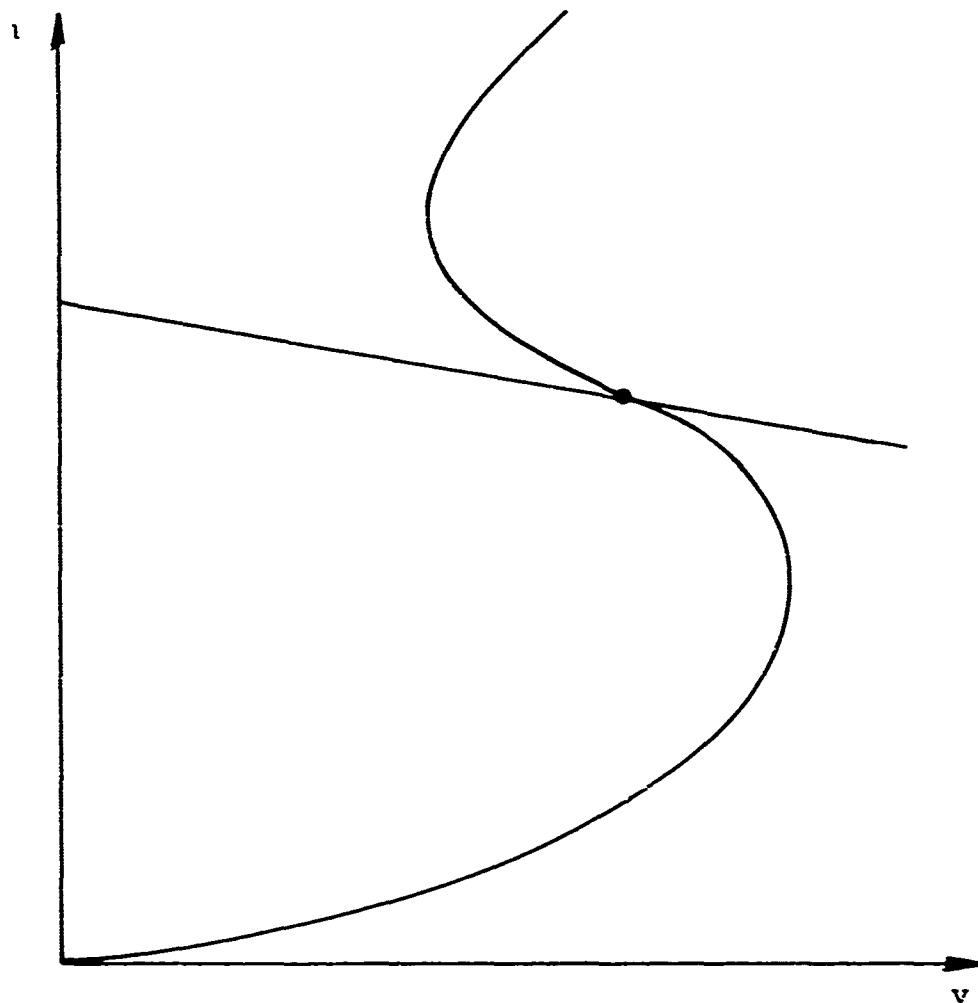


Figure 9-3. Typical Current/Voltage Characteristic Curve for a Current-Controlled Negative-Resistance Device with One Intersection by a Load Line

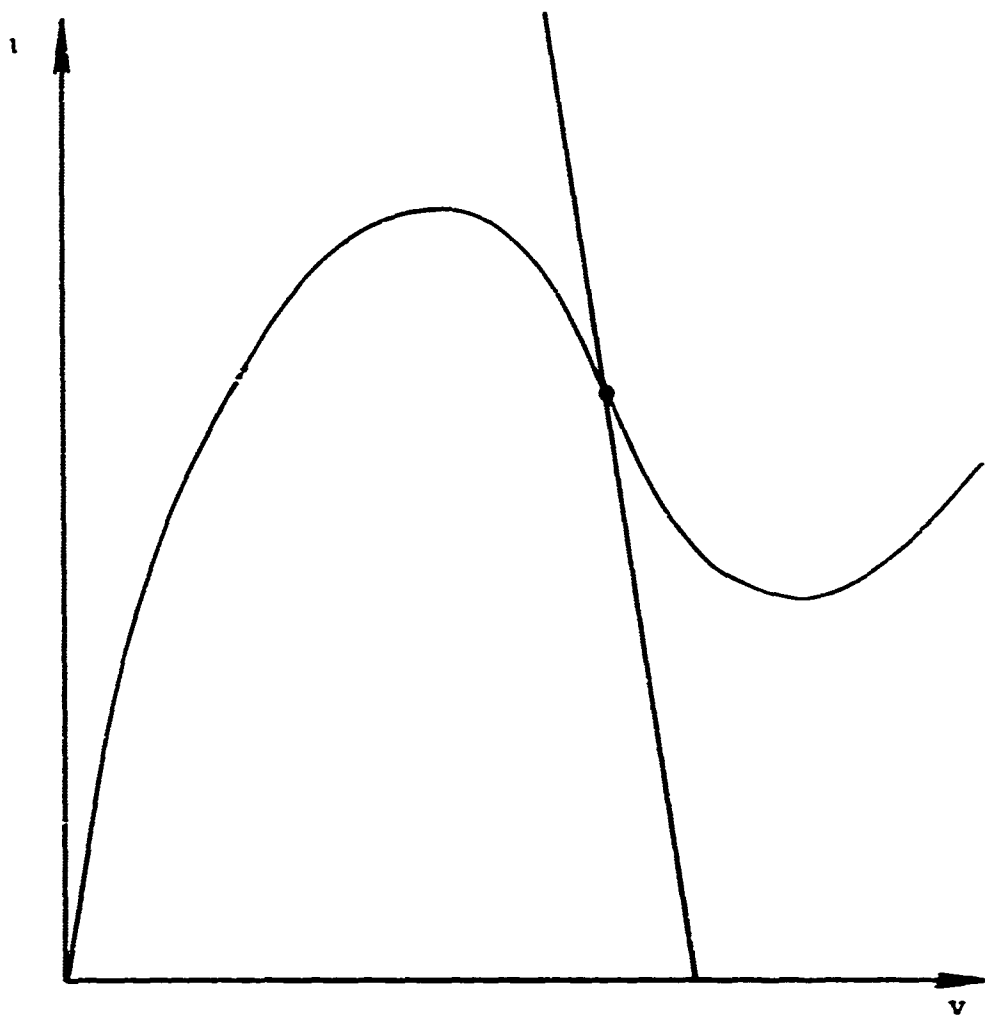


Figure 9-4. Regular Current/Voltage Characteristic Curve for a Voltage-Controlled Negative-Resistance Device with One Intersection by a Load Line



## SECTION X

### ESSENTIAL ELEMENT IN THE EQUIVALENT CIRCUIT

Only one of the energy storage elements in the dynamic equivalent circuit for a negative-resistance device is essential to approximate the behavior of the device except when it operates as an oscillator. If the exception is not considered, the behavior of the system can be described by a single first-order linear differential equation.

If the equivalent circuit for the current-controlled negative-resistance device did not contain any shunt capacitance, the equation describing the behavior of the device can be written as

$$\frac{di}{dt} + \frac{1}{L_c} [r_c + R_c] i(t) = 0 \quad (10-1)$$

The single characteristic root for equation 10-1 is

$$s = -\frac{1}{L_c} [r_c + R_c]$$

In the negative slope region of the S-type curve as shown in Figure 8-1, the load resistance is less than the magnitude of the incremental resistance of the device.

$$R_c < |r_c|$$

The value of the incremental resistance in the negative-slope region is less than zero.

$$r_c < 0$$

Consequently, the characteristic root is greater than zero.

$$s > 0$$

The solution to equation 10-1 is subjected to an exponential growth. Therefore, the operation of the system is unstable. In the positive slope portions of the S-type curve as shown in Figure 8-1, the value of the incremental resistance is greater than zero.

---

The essential element in the equivalent circuit to predict the behavior of the voltage-controlled negative-resistance device is discussed by Cunningham [12].

$$r_c > 0$$

Therefore, the characteristic root for equation 10-1 is less than zero.

$$s < 0$$

The solution to the differential equation 10-1 undergoes an exponential decay. Therefore the operation of the system is stable. These results agree theoretically with those already found with both reactive elements present in the equivalent circuit for the device.

If the equivalent circuit for the current-controlled negative-resistance device did not contain any series inductance, the equation describing the behavior of the device can be written

$$\frac{di}{dt} + \frac{1}{C_c} \left[ \frac{1}{r_c} + \frac{1}{R_c} \right] i(t) = 0 \quad (10-2)$$

The single characteristic root for equation 10-2 is

$$s = -\frac{1}{C_c} \left[ \frac{1}{r_c} + \frac{1}{R_c} \right]$$

In the negative-slope region of the S-type curve as shown in Figure 8-1, the load resistance is less than the magnitude of the incremental resistance of the device. Since the value of the incremental resistance in the negative-slope region is less than zero, the characteristic root is less than zero. The solution to the differential equation 10-2 is subjected to an exponential decay. Consequently, the operation of the system is stable. In the positive-slope portions of the S-type curve the value of the incremental resistance is greater than zero. Therefore, the characteristic root for equation 10-2 is less than zero. The solution to the differential equation 10-2 undergoes an exponential decay. Consequently, the operation of the system is stable. These results do not agree theoretically with those already found with both energy storage elements present in the equivalent circuit for the device.

From the conclusion of the preceding paragraphs, it appears that the series inductor in the equivalent circuit for the current-controlled negative-resistance device is the essential element in obtaining the proper behavior of the device. Similar reasoning pertains to the voltage-controlled negative-resistance device, where the shunt capacitor in the equivalent circuit for the device seems to be the essential element to obtain the proper behavior.

## SECTION XI

### CONDITIONS FOR STABILITY

An unstable point of operation is one where an infinitesimal fluctuation of current or voltage is sufficient to cause the operating point to shift automatically to a stable one.

In the neighborhood of a simple singularity, the stability behavior of a non-linear system is determined by the approximate linear differential equation which describes the operation of the system. The general solution to the linear second-order differential equation for the small signal equivalent circuit of the current-controlled negative-resistance device is

$$i(t) = A e^{st}$$

If  $A e^{st}$  is substituted for  $i(t)$  in equation 9-4, the following quadratic characteristic algebraic equation is obtained

$$s^2 + s \left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right] + \frac{1}{L_c C_c} \left[ 1 + \frac{r_c}{R_c} \right] = 0 \quad (11-1)$$

The two roots for equation 11-1 are

$$s = -\frac{1}{2} \left\{ \left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right] \pm \sqrt{\left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right]^2 - \frac{4}{L_c C_c} \left[ 1 + \frac{r_c}{R_c} \right]} \right\}$$

Therefore, two independent solutions exist for equation 9-4. A solution can have the form

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Some of the worthy references which relate the different stability conditions to the operation of the negative resistance device are Hinds (15), Minorsky (22) and Swamy (30).

where

$$S_1 = -\frac{1}{2} \left\{ \left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right] + \sqrt{\left( \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right)^2 - \frac{4}{L_c C_c} \left( 1 + \frac{r_c}{R_c} \right)} \right\}$$

$$S_2 = -\frac{1}{2} \left\{ \left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right] - \sqrt{\left( \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right)^2 - \frac{4}{L_c C_c} \left( 1 + \frac{r_c}{R_c} \right)} \right\}$$

where  $A_1$  and  $A_2$  are arbitrary constants depending upon the initial current in the inductor and initial charge on the capacitor. The exponential factors  $S_1$  and  $S_2$  which depend upon the choice of circuit parameters may be real, complex, or imaginary.

The effect of the circuit parameters  $L_c$ ,  $C_c$ ,  $R_c$ , and  $r_c$  on the stability of the current-controlled negative-resistance device is studied by considering three two-dimensional cross sections  $(r_c, R_c)$ ,  $(r_c, L_c)$  and  $(r_c, C_c)$ . The parameters  $R_c$ ,  $L_c$  and  $C_c$  may have only positive values, whereas the parameter  $r_c$  may have positive and negative values which are determined by the location of the operating point on the S-type curve.

The different stability regions in the  $(r_c, R_c)$  plane are shown in Figure 11-1.

The roots of the characteristic algebraic equation can be written

$$S = -\frac{L_c + R_c C_c r_c}{2R_c L_c C_c} \pm \frac{1}{2R_c L_c C_c} \sqrt{L_c^2 + R_c^2 C_c^2 r_c^2 - 2R_c L_c C_c r_c - 4L_c R_c C_c}$$

The quantity under the square root has the form

$$\left[ L_c - R_c C_c r_c \right]^2 - \left[ 2R_c \sqrt{L_c C_c} \right]^2$$

The condition for complex roots is that the quantity under the square root is less than zero.

$$\left[ L_c - R_c C_c r_c \right]^2 - \left[ 2R_c \sqrt{L_c C_c} \right]^2 < 0$$

The following two equations are considered:

$$L_c - R_c C_c r_c + 2R_c \sqrt{L_c C_c} = 0 \quad (11-2)$$

$$L_c - R_c C_c r_c - 2R_c \sqrt{L_c C_c} = 0 \quad (11-3)$$

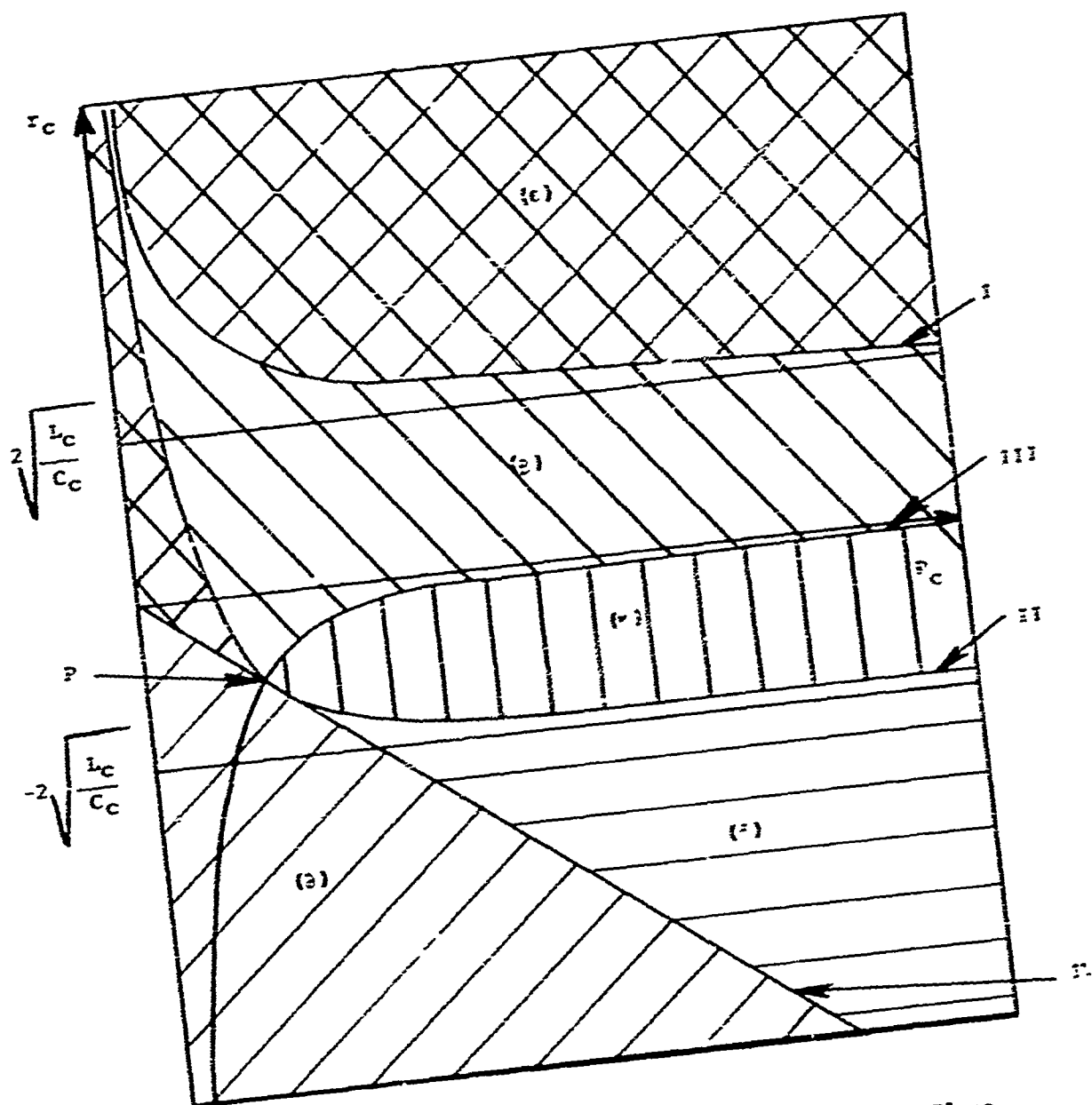


Figure 11-1. Different Stability Regions in the  $(\alpha_c, R_c)$  Plane

Equations 11-2 and 11-3 represent hyperbolas in the  $(r_c, R_c)$ -plane as shown by curves I and II in Figure 11-1. The  $r_c$  axis is an asymptote for both hyperbolas. The line  $r_c = 2\sqrt{L_c/C_c}$  is asymptotic to curve I and the line  $r_c = -2\sqrt{L_c/C_c}$  is asymptotic to curve II. Therefore, the area between hyperbolas I and II is the region where the complex roots of equation 11-1 are located. Since the complex roots correspond to focal points, it is deduced that the focal points are situated in the portion between the two hyperbolas. These focal points are unstable if

$$L_c + R_c C_c r_c < 0$$

and stable if

$$L_c + R_c C_c r_c > 0$$

The equation

$$L_c + R_c C_c r_c = 0$$

represents a hyperbola with  $r_c$  and  $R_c$  axes as asymptotes as shown by curve III in Figure 11-1. Consequently the region (B) between curves I and II, but above curve III, corresponds to stable focal points. The region ( $\alpha$ ) between curves II and III to the right of their intersection corresponds to unstable focal points. Hence, there is a region  $\alpha$ , where self-excitation of oscillation is possible because the oscillations increase on a divergent spiral starting from an unstable focal point. It is observed that in a region B, where  $r_c > 0$ , self-excitation of oscillation is not possible since there are not any unstable focal points in this region.

It is seen that saddle points arise if

$$R_c + r_c < 0$$

Therefore, the area ( $\gamma$ ) below the straight line IV

$$R_c + r_c = 0$$

corresponds to the region of saddle points, and is a region of instability. Physically, this means that energy is supplied to the system from an external generator at a rate greater than its rate of dissipation.

It is observed that a point P at the intersection of curves II and III has coordinates  $+\sqrt{L_c/C_c}$  and  $-\sqrt{L_c/C_c}$ , and the straight line IV passes through this point. The region of focal points is generally separated from that of saddle points by an intermediate region of nodal points. It is inferred that the region between curve II and straight line IV is a zone of nodal points. The region ( $\epsilon$ ) of nodal points to the left of and above P corresponds to stable nodal points. The region ( $\delta$ ) to the right of and below P contains

the unstable nodal points. Consequently, P is a point of separation for the various types of singularities.

The various stability portions in the  $(r_c, L_c)$  plane are shown in Figure 11-2.

The bounds of the area of distribution of the complex roots or focal points is given by the following equation:

$$L_c^2 + [R_c C_c r_c]^2 - 2 R_c C_c r_c - 4 L_c C_c R_c^2 = 0 \quad (11-4)$$

The succeeding two equations are studied:

$$r_c = \frac{L_c}{R_c C_c} + 2 \sqrt{\frac{L_c}{C_c}} \quad (11-5)$$

$$r_c = \frac{L_c}{R_c C_c} - 2 \sqrt{\frac{L_c}{C_c}} \quad (11-6)$$

Equation 11-5 represents curve I and equation 11-6 represents curve II in the  $(r_c, L_c)$ -plane as shown in Figure 11-2. Equation 11-4 is a parabola passing through  $(0, 0)$  and is represented by curve I and curve II. This parabola has a vertical tangent at the origin and a horizontal tangent at  $(-R_c, R_c^2 C_c)$ . Therefore, the focal points are situated inside of this parabola. The stable and unstable focal points may be separated by drawing the straight line III

$$L_c + C_c R_c r_c = 0$$

which passes through the origin with a slope of  $-1/R_c C_c$ . The regions ( $\alpha$ ) and (B) respectively correspond to the regions of unstable and stable focal points. Consequently, self-excited oscillations are possible only for negative values of  $r_c$  and for not too large values of  $L_c$ . The zone of saddle points is obtained by drawing the straight line

$$R_c + r_c = 0$$

as shown by curve IV in Figure 11-2.

The region ( $\gamma$ ) below curve IV is the zone of saddle points. Between straight line IV and curve II but to the left of their intersection is the region ( $\delta$ ) of unstable nodal points. Between straight line IV and curve II but to the right of their intersection is the zone ( $\epsilon$ ) of stable nodal points.

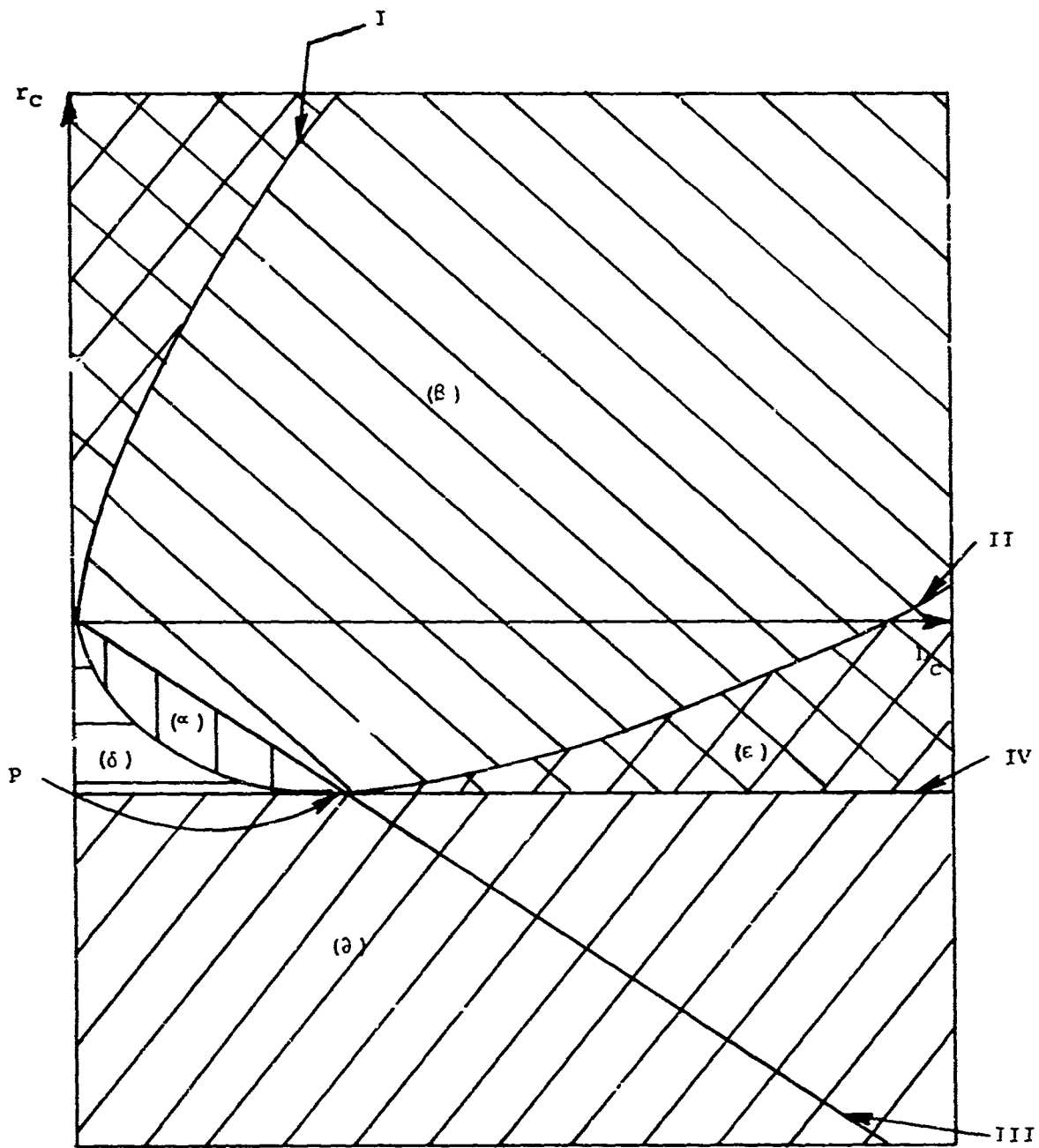


Figure 11-2. Different Stability Regions in the  $(r_c, L_c)$  Plane



The region ( $\gamma$ ) below curve IV is the zone of saddle points. Between straight line IV and curve II but to the left of their intersection is the region ( $\delta$ ) of unstable nodal points. Between straight line IV and curve II but to the right of their intersection is the zone ( $\epsilon$ ) of stable nodal points.

The different stability zones in the ( $r_c$ ,  $C_c$ ) plane are shown in Figure 11-3.

The limits of the region of distribution of the complex roots or focal points is specified by the succeeding equation:

$$L_c^2 + \left[ R_c C_c r_c \right]^2 - 2 R_c L_c C_c r_c - 4 L_c C_c R_c^2 = 0$$

The following two equations are considered:

$$L_c - R_c C_c r_c + 2 R_c \sqrt{L_c C_c} = 0 \quad (11-7)$$

$$L_c - R_c C_c r_c - 2 R_c \sqrt{L_c C_c} = 0 \quad (11-8)$$

Equation 11-7 describes curve I and equation 11-8 describes curve II in the ( $r_c$ ,  $C_c$ ) -plane as shown in Figure 11-3. The  $r_c$  and  $C_c$  axes are asymptotes to the hyperbolic curve I. Curve II has the  $r_c$  axis as an asymptote and a horizontal tangent at  $(-R_c, L_c/R_c^2)$ . Consequently, the focal points are located between curve I and curve II. The stable and unstable focal points may be divided by sketching the equation

$$L_c + C_c R_c r_c = 0$$

which is represented by hyperbolic curve III. The  $-r_c$  and  $C_c$  axes are asymptotic to curve III. The zones ( $\alpha$ ) and ( $\beta$ ) respectively match to the areas of unstable and stable focal points. Thus, self-excited oscillations are possible for negative values of  $r_c$  and for not too small values of  $L_c$ . The region of saddle points is obtained by drawing the straight line

$$R_c + r_c = 0$$

as shown by curve IV in Figure 11-3. The region below curve IV is the area of saddle points. The zone between curves II and IV but to the right of their intersection corresponds to unstable nodal points. The area between curve I and IV but to the left of their intersection corresponds to stable nodal points.

The two-dimensional cross sections ( $r_c$ ,  $R_c$ ), ( $r_c$ ,  $L_c$ ) and ( $r_c$ ,  $C_c$ ) help establish general criteria for the stability of operation for the current-controlled negative-resistance device. A load line which cuts the S-type curve at three points as shown in Figure 8-1 is considered. For the intersections in the positive slope regions, the

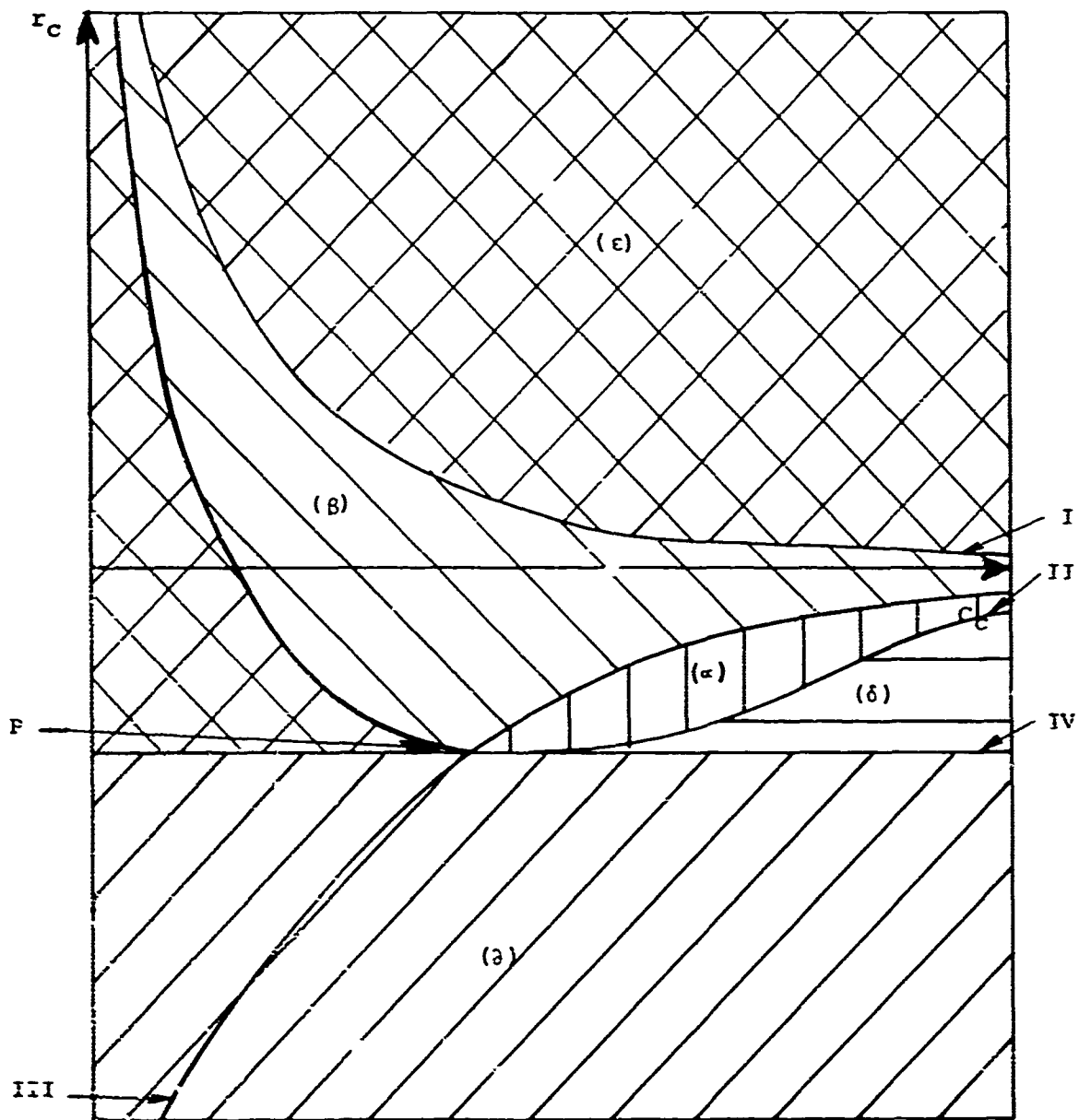


Figure 11-3. Different Stability Regions in the  $(r_c, C_c)$  Plane

incremental resistance is positive. From the Figures 11-1, 11-2, and 11-3, it is seen that the singularities in the upper half plane are all stable. Therefore, the intersections in the positive-slope regions are stable equilibrium points. For the intersection in the negative-slope region, the magnitude of the incremental resistance is greater than  $R_C$ . Consequently the singularities lie below the curve IV in the  $(r_C, R_C)$ ,  $(r_C, L_C)$  and  $(r_C, C_C)$  planes which is a region ( $\gamma$ ) of saddle points. Thus, the intersection in the negative-resistance region is an unstable equilibrium point.

A load line which intersects the current/voltage characteristic curve for the current-controlled negative-resistance device at one point in the negative-slope region as shown in Figure 9-3 is studied. For this type of intersection,  $R_C$  is greater than the magnitude of  $r_C$ . Therefore, the singularities are above the curve IV in the Figures 11-1, 11-2, and 11-3. Consequently, the singularities cannot be saddle points. It is seen that these singularities may be stable or unstable focal points, or may be stable or unstable nodal points depending on the parameters  $R_C$ ,  $L_C$ ,  $C_C$ , and  $r_C$ . There is a region ( $\beta$ ) of absolute stability (stable focal points) which implies decaying oscillations for any disturbance. This region may be used for tuned amplifier circuits. There is a zone ( $\alpha$ ) of conditional stability (unstable focal points) which indicates growing oscillations for some disturbance. This zone may be applied to oscillator circuits. There is an area ( $\epsilon$ ) of stable nodal points which implies an exponential decay for any disturbance. This area may be used for untuned amplifier circuits. There is a region ( $\delta$ ) of unstable nodal points which indicates an exponential growth for some disturbance. The previous study points out that the zone of absolute instability (saddle points) starts when  $|r_C| = R_C$  and sometimes absolute instability begins when  $|r_C| < R_C$  in the region of unstable nodal points. It is evident that for a fixed value of  $r_C < 0$ , the stability condition of a given static point  $(r_C, R_C)$  is also affected by the two other parameters  $L_C$  and  $C_C$ . This investigation has taken into consideration both the static and dynamic elements of equilibrium.

By the principle of duality, the effect of the circuit parameters  $G_V$ ,  $C_V$ ,  $L_V$ , and  $g_V$  on the stability of the voltage-controlled negative-resistance device is considered by studying three two-dimensional cross sections  $(g_V, G_V)$ ,  $(g_V, C_V)$  and  $(g_V, L_V)$ .

The relations among the coefficients of equation 11-1 for each type of singularity are shown in Table 11-1.

TABLE 11-1. RELATIONS AMONG COEFFICIENTS FOR  
EACH TYPE OF SINGULARITY

Saddle:  $\frac{4}{L_c C_c} \left[ 1 + \frac{r_c}{R_c} \right] < 0$

Both roots are real and of opposite sign.

Node:  $\left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right]^2 > \frac{4}{L_c C_c} \left[ 1 + \frac{r_c}{R_c} \right] > 0$

Both roots are real and of same sign.

Focus:  $\frac{4}{L_c C_c} \left[ 1 + \frac{r_c}{R_c} \right] > \left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right]^2$

Roots are complex conjugates.

For node or focus

Stable:  $\left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right] < 0$

If the real part of the characteristic root is negative, the solution is stable.

Unstable:  $\left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right] > 0$

If the real part of the characteristic root is positive, the solution is unstable.

Vortex:  $\left[ \frac{r_c}{L_c} + \frac{1}{R_c C_c} \right] = 0 + \frac{4}{L_c C_c} \left[ 1 + \frac{r_c}{R_c} \right] > 0$

Both roots are pure imaginary. The solution has neutral stability.

## SECTION XII

### LIMIT CYCLE

The mechanism of behavior of a negative-resistance device produces a time lag between the excitation (cause) and the responses (effect). This time lag causes the instantaneous operating point to describe an ellipse (limit cycle) on the static current, voltage characteristic curve as shown in Figures 12-2 and 12-3. This limit cycle appears only if certain conditions are satisfied.

An interesting case is considered when a current-controlled negative-resistance device is driven from a source that has a high internal resistance. The parameters are chosen to make  $|r_c| > L_c R_c C_c$ , so that the magnitude of the incremental resistance in the negative-slope region is greater than the impedance of the series  $R_c L_c C_c$  circuit at its resonant frequency as shown in Figure 9-1. Therefore, the two characteristic roots

$$S_1 = -\frac{1}{2} \left\{ \left[ \frac{1}{R_c C_c} + \frac{r_c}{L_c} \right] - \sqrt{\left[ \frac{1}{R_c C_c} + \frac{r_c}{L_c} \right]^2 - \frac{4}{L_c C_c} \left[ 1 - \frac{r_c}{R_c} \right]} \right\}$$

$$S_2 = -\frac{1}{2} \left\{ \left[ \frac{1}{R_c C_c} - \frac{r_c}{L_c} \right] - \sqrt{\left[ \frac{1}{R_c C_c} - \frac{r_c}{L_c} \right]^2 - \frac{4}{L_c C_c} \left[ 1 - \frac{r_c}{R_c} \right]} \right\}$$

are sure to have a positive real part in the negative-slope region, and the operation of the device is unstable. If the loss resistance is greater than the magnitude of the incremental resistance,

$$R_c > |r_c|$$

the singularity cannot be a saddle point. Consequently, the singularity must be either a nodal point or a focal point. If

$$\frac{4}{L_c C_c} \left[ 1 - \frac{r_c}{R_c} \right] > \left[ \frac{r_c}{L_c} - \frac{1}{R_c C_c} \right]^2$$

The behavior of the negative-resistance device which is operated as an oscillator describes a limit cycle. This is considered by Cunningham 12, Herold 14, Lynn 15 and Vickers 33.

the singularity is a focal point which indicates the production of oscillations. For the virtual singularities as shown in Figure 12-1, the incremental resistance is greater than zero. Therefore, the characteristic roots must have negative real parts which represent stable operation of the device. These singularities are stable focal points which imply the creation of decaying oscillations.

A probable set of solution curved lines is drawn in Figure 12-1. It is assumed that an unstable focal point exists when the singularity is in the negative-slope region while a stable focal point exists when the singularity is in either positive-slope region. The solution curves which relate the instantaneous values of the current and voltage at the terminals of the device are spiral curves diverging from the singularity in the negative-slope region. When the solution curves cross into the positive-slope regions, they become spiral curves converging toward the virtual singularities as shown in Figure 12-1. A resultant elliptical curve is eventually reached which is known as a limit cycle.

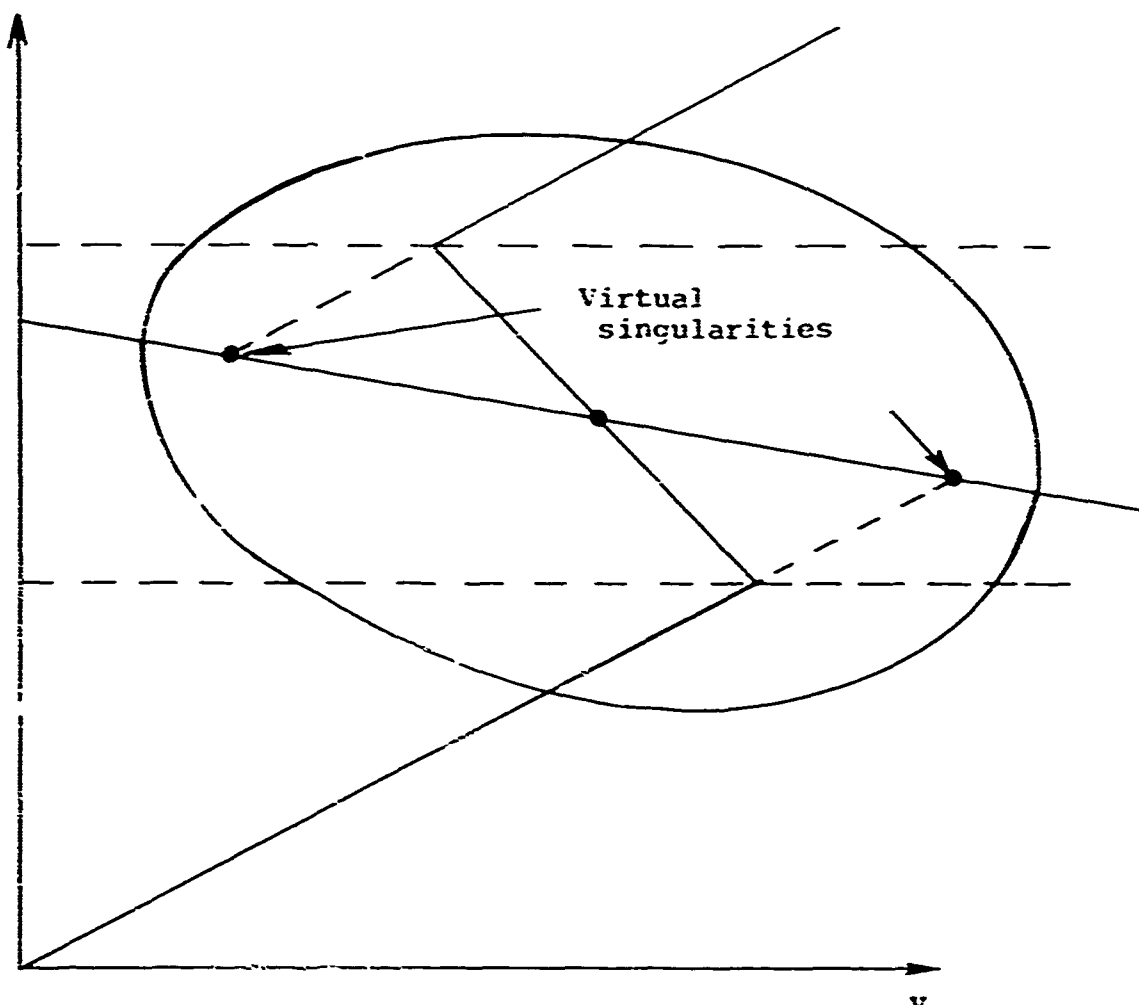


Figure 12-1. Piece Wise Continuous Characteristic Curve for a Current-Controlled Negative-Resistance Device with One Intersection by a Load Line

It is obvious from Figure 12-1 that the limit cycle consists of portions of expanding curves which are associated with the singularity in the negative-slope region and contracting curves which are associated with the virtual singularities in the positive-slope regions. The limit cycle represents a periodic and usually non-sinusoidal oscillation. The amplitude and the waveform of the oscillation depend only on the parameters of the system and do not depend on the initial conditions which start the oscillation.

The existence of the limit cycle can be explained in the following way: If initially the amplitude is extremely small, the solution curve is completely in the negative-slope region on the S-type curve. Therefore, the amplitude must grow. Alternately, if initially the amplitude is exceedingly large, the solution curve is mostly in the positive-slope regions. Consequently, the amplitude must decay. There must be some intermediate amplitude that can neither grow nor decay. Therefore, the result is reached that the operation of the above system must come to a steady-state oscillation of constant amplitude and mean value. This operation is described by the limit cycle as shown in Figure 12-2. In the current-controlled negative-resistance device, the sense of the rotation of the ellipse is counterclockwise.

Similarly, by the principle of duality, when a voltage-controlled negative-resistance device is driven from a generator that has a low internal resistance and the other conditions are satisfied, a limit cycle appears on the N-type characteristic curve as shown in Figure 12-3. In this device the sense of rotation of the ellipse is clockwise.

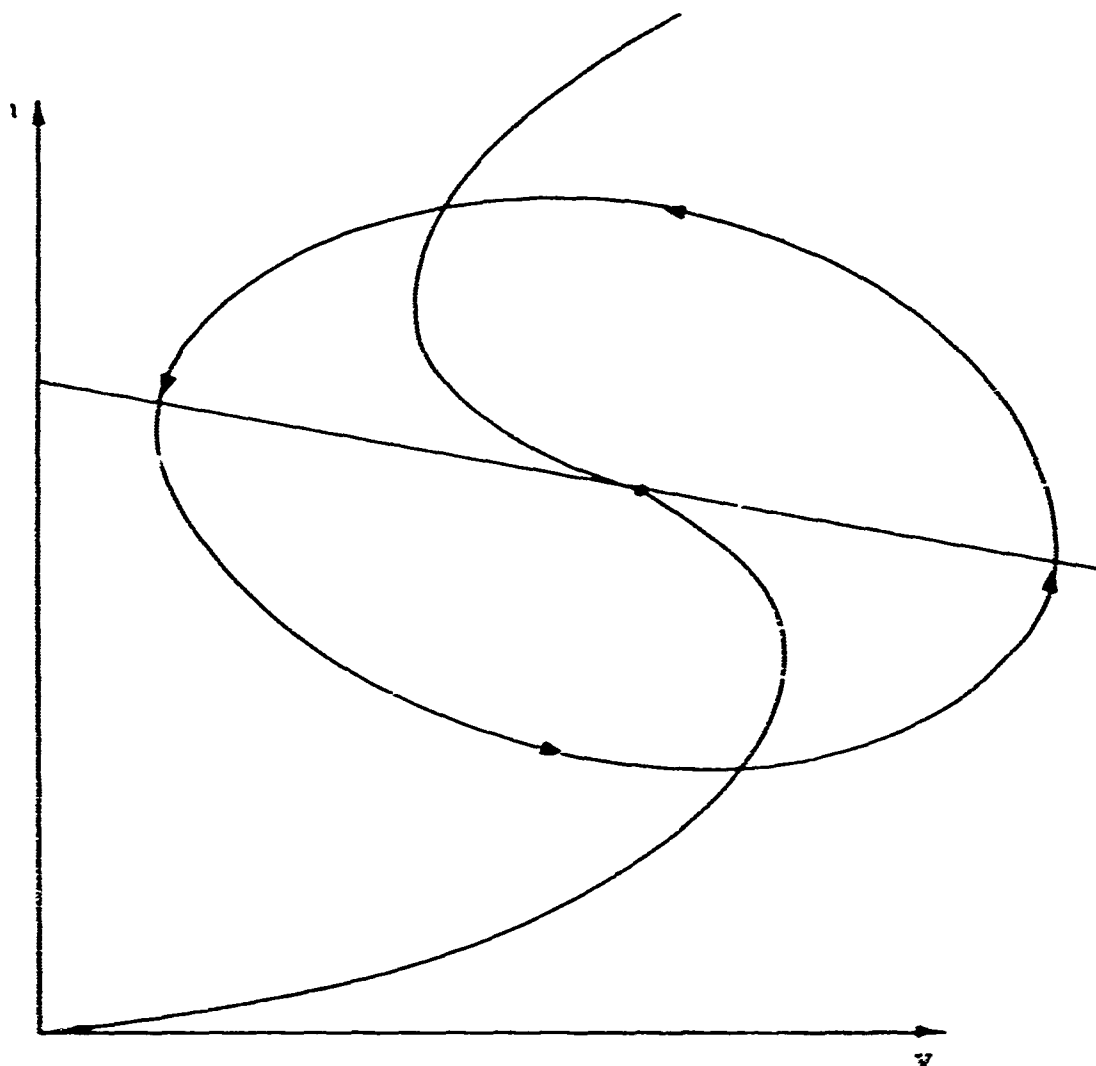


Figure 12-2. Typical Static Characteristic Curve for a Current-Controlled Negative-Resistance Device with its Dynamic Limit Cycle Curve in Counterclockwise Rotation



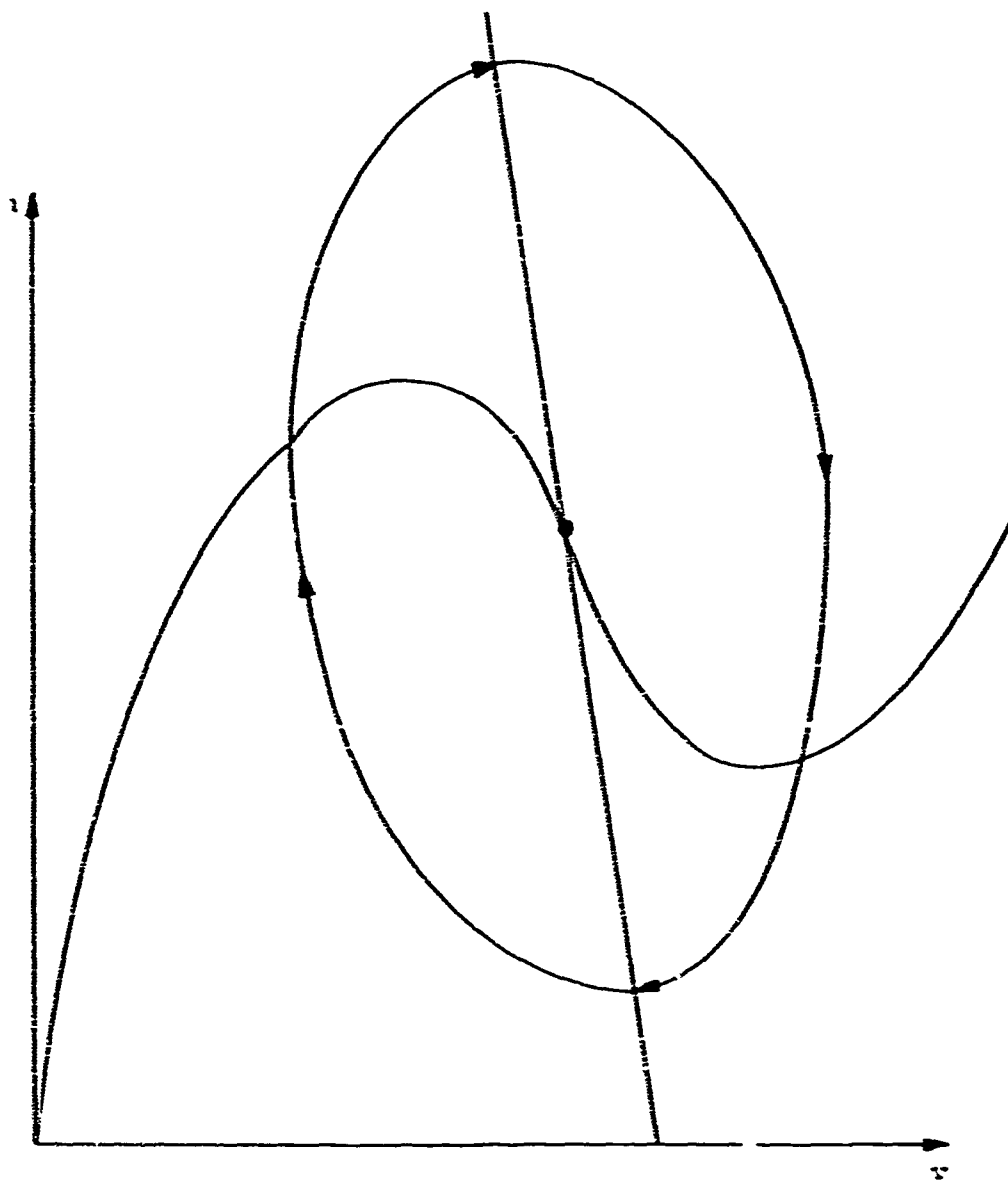


Figure 12-3. Typical Static Characteristic Curve for a Voltage-Controlled Negative-Resistance Device with its Dynamic Limit Cycle Curve in Clockwise Rotation

## SECTION XIII

### THEORY FOR THE CURRENT-CONTROLLED NEGATIVE-RESISTANCE DEVICE USING THE GLOW DISCHARGE TUBE FOR AN EXAMPLE

The glow discharge which makes use of the cold cathode is distinguished by a moderately high voltage drop across the terminals of the tube and a low current through the terminals. There is a region in the operating range of the glow discharge tube where the voltage drop between the terminals decreases while the current through the terminals increases. The current/voltage characteristic curve for the tube as shown in Figure 13-1 shows that the breakdown voltage is slightly higher than the maintaining voltage for the discharge. Consequently, there is a region where the tube has a negative resistance. Above the negative-slope region, there is a portion of the characteristic curve which has a constant voltage for a small range of currents.

The mechanism of behavior of the glow discharge tube is considered. At zero plate voltage, a few of the electrons have enough energy to reach the plate. Therefore, the current through the terminals of the tube should be small. When the static plate voltage is increased, the electrons will drift toward the plate. If the electric field is sufficiently large, the free electrons may obtain enough energy to ionize the gas molecules when the electrons and molecules collide with each other. In the applied electric field, the electrons move toward the plate and the positive ions move toward the cathode. If the field is quite large, the resulting ionization may last until breakdown happens. When the breakdown takes place, the voltage distribution within the tube is noticeably modified and a self-sustaining discharge is established. Each electron which is released from the cathode acquires enough energy on the way to the plate to produce a large number of positive ions. When these positive ions, on the average, arrive back at the cathode and release another electron, the self-sustaining condition has been achieved. A space charge of positive ions begins to develop close to the cathode. The region between the cathode and the plate may be divided into two major portions which are the cathode fall and the plasma. The cathode fall which has a positive ion density is a thin zone near the cathode. Most of the voltage drop between the plate and the cathode happens in this zone. Therefore, the greatest portion of the electric field appears between the cathode and the positive ions in the cathode fall region. The freed electrons are accelerated through this region. Practically all of the ionization of the gas molecules occurs in the cathode fall. The plasma is a very wide zone which has, at any instant of time, the number of electrons almost equal to the number of positive

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The theory for the current-controlled negative-resistance device (glow discharge tube) is explained by Benson (3-6), Chirlian and Zemanian (10), Mac Dougall (19), Milman and Seely (21), Townsend and Depp (32) as well as Yeh and Chaffee (34). The impedance locus of tunnel diodes is investigated by Card (8).

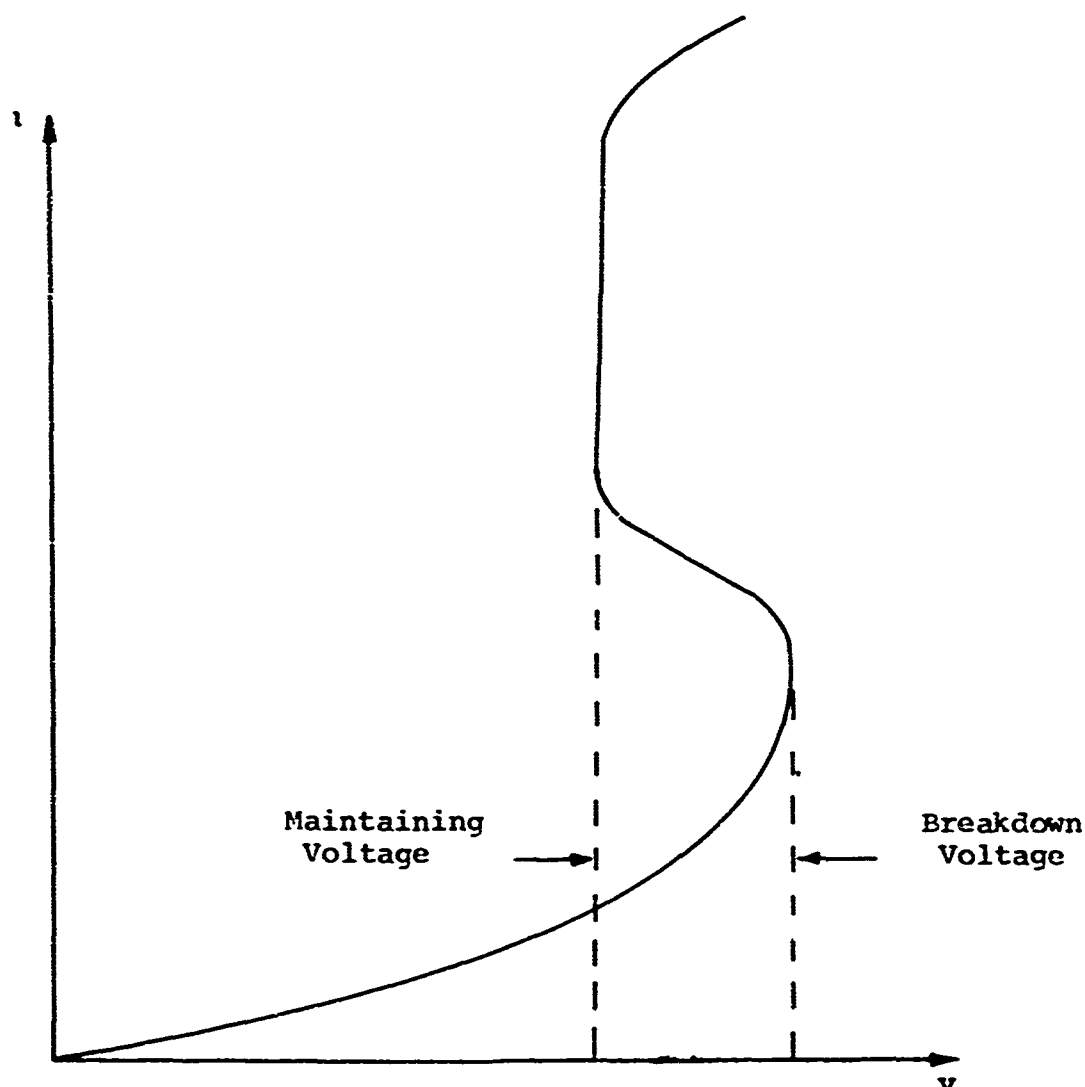


Figure 13-1. Current/Voltage Characteristic Curve for Glow Discharge Tube

ions. The electrons drift faster toward the plate than the heavier positive ions drift toward the cathode. Therefore the electron current is much larger than the positive ion current in the plasma. This zone behaves as an extension of the plate. Consequently, the plasma helps to reduce the effective distance between the cathode and the plate. When the plasma is present, a smaller voltage drop between the cathode and the plate is needed to produce the same electric field near the cathode. During the formation of the plasma and the cathode fall, the secondary emission of electrons at the cathode increases which is due to the positive ion bombardment. Therefore, the ionization of the gas atoms is made easier and more charged particles are produced. This permits the voltage across the terminals of the tube which is necessary to maintain the discharge to decrease, while the current through the terminals increases. The voltage continues to decrease until the formation of the plasma and the cathode

fall is completed. At this point, the discharge is distinguished by the fact that the current through the terminals increases while the voltage remains constant.

When a small alternating signal is superimposed on the static bias of a glow discharge tube, the magnitude of the experimental input impedance is found to first increase, then to decrease with increasing frequency. The impedance has an inductive reactance at audio frequencies, but has a capacitive reactance at frequencies greater than 1 MHz as shown in Figure 13-2. Measurements to determine the impedance/frequency characteristics of various glow discharge tubes which are biased in the negative-slope region showed that the effective inductance decreased and the effective resistance increased with an increase in frequency from 300 Hz - 30 KHz. The inductance and resistance decreased with an increase in current through the terminals of the tube.

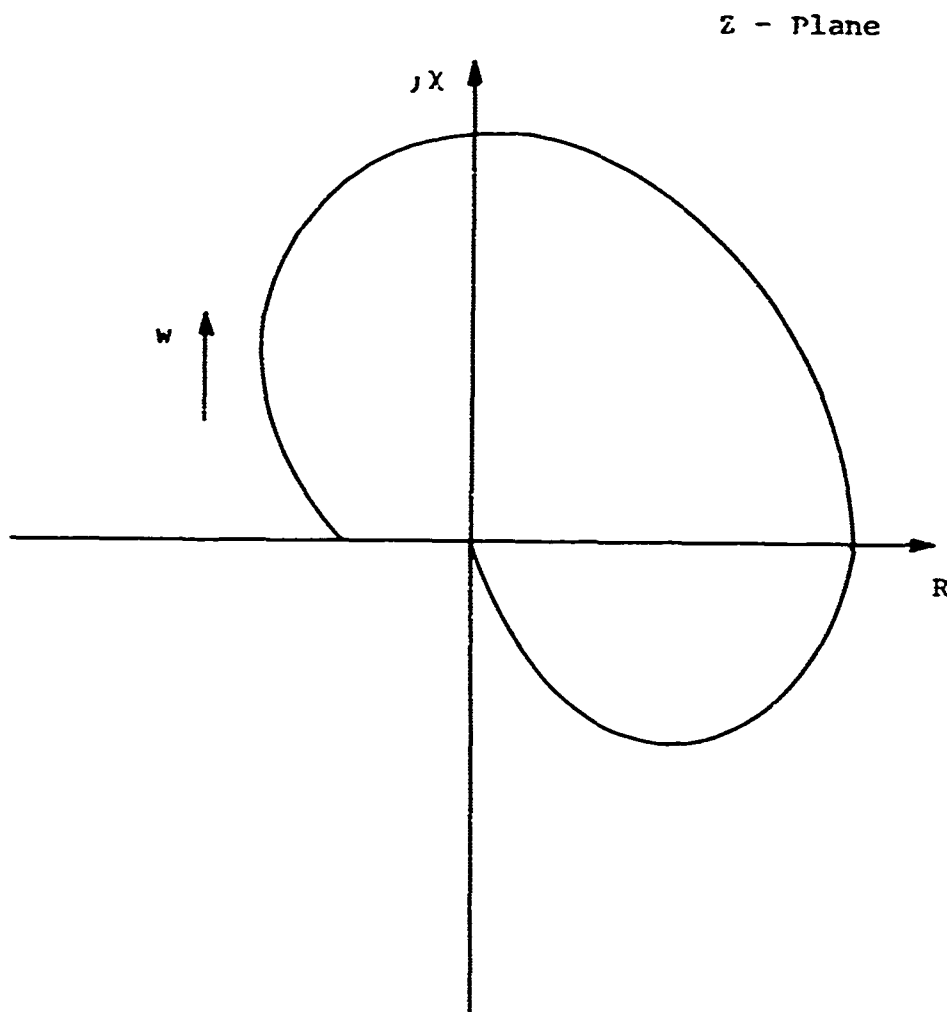


Figure 13-2. Experimental Reactance/Resistance Characteristic for the Input Impedance of a Glow Discharge Tube Operated in the Negative-Resistance Region

A theory that was derived by Van Geel studied small alternating signals which are superimposed on a static bias condition. It is assumed that the physical quantities in the discharge are functions of the instantaneous as well as delayed current and voltage. This hypothesis predicted that the impedance loci should be semi-circles on the reactance/resistance characteristic curve (neglecting the effective capacitance and effective resistance) as shown in Figure 13-3. This theory postulated the form of the impedance locus when a secondary mechanism having a delay time is present in the discharge.

The input impedance locus of a glow discharge tube which is operated in the negative slope region has the same form as the theory that was predicted by Van Geel if it is extended to include several delayed effects. There are secondary processes of electron production in most discharges which can be regarded as delayed functions of current and voltage, because these processes are associated with a delay time. Diffusion processes in the discharge also can be treated as delayed effects. Secondary emission and diffusion will influence the form of the input impedance of the glow discharge tube. Consequently, any process which influences the delay of current and voltage and has a time constant associated with it will be handled as a delayed effect.

Z - Plane

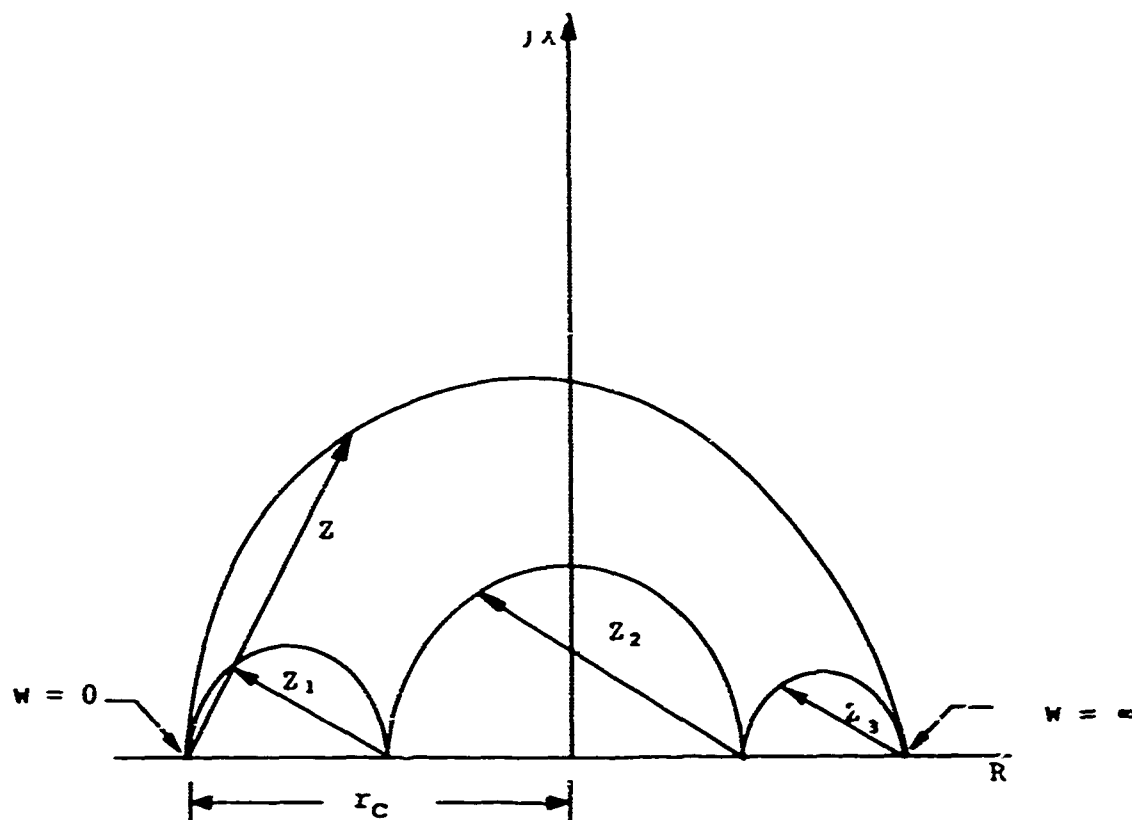


Figure 13-3. Theoretical Reactance/Resistance Characteristic for the Input Impedance of a Glow Discharge Tube Operated in the Negative-Resistance Region

The theoretical input impedance of a glow discharge tube which is operated in the negative slope region is derived by considering the behavior of  $n$ -delayed effects. The impedance has the following form

$$Z_c(j\omega) = R_o - \sum_{k=1}^{k=n} \frac{R_k}{1 + j\omega\tau_k} \quad (13-1)$$

where  $R_o, R_1, \dots, R_k, \dots, R_n$  are resistances and  $\tau_1, \tau_2, \dots, \tau_k, \dots, \tau_n$  are time constants associated with the various delay processes. The second term of equation 13-1 involves the contribution from the secondary mechanisms in the discharge process. The quantity  $R_o$  in equation 13-1 is given by

$$R_o = r_c + R_1 + R_2 + \dots + R_n$$

where  $r_c$  is the reciprocal of the slope of the static current/voltage characteristic curve.

Consequently, the theoretical input impedance for  $n$ -delayed effects can be written

$$Z_c(j\omega) = r_c + \frac{j\omega L_1}{1 + j\omega L_1/R_1} + \dots + \frac{j\omega L_n}{1 + j\omega L_n/R_n} \quad (13-2)$$

where  $L_1, L_2, \dots, L_n$  are inductances associated with the various delay processes and

The second and subsequent terms of equation 13-2 represent a parallel combination of an inductor  $L_n$  and a resistor  $R_n$ . The basic equivalent circuit can be identified from the form of the theoretical input impedance as shown in Figure 13-4. It can be represented by a series combination of the incremental resistance and a number of combinations of inductors and resistors. The actual equivalent circuit also includes the effective capacitance  $C_c$ , the effective resistance  $R_d$ , and the load resistance  $R_c$  as shown in Figure 13-5. Each parallel combination of  $L_n$  and  $R_n$  has a time constant associated with a particular process in the discharge mechanism.

Investigations show that the experimental impedance locus of the glow discharge tube is not a semi-circle. Instead, the actual locus is a circular type locus as shown in Figure 13-3. This is due to the presence of the effective capacitance and the effective resistance.

Measurements showed that the effective capacitance of all glow discharge tubes approaches a limiting value at frequencies above 4 MHz. Consequently, it appeared to be independent of the frequency above 4 MHz. This capacitance was found to be

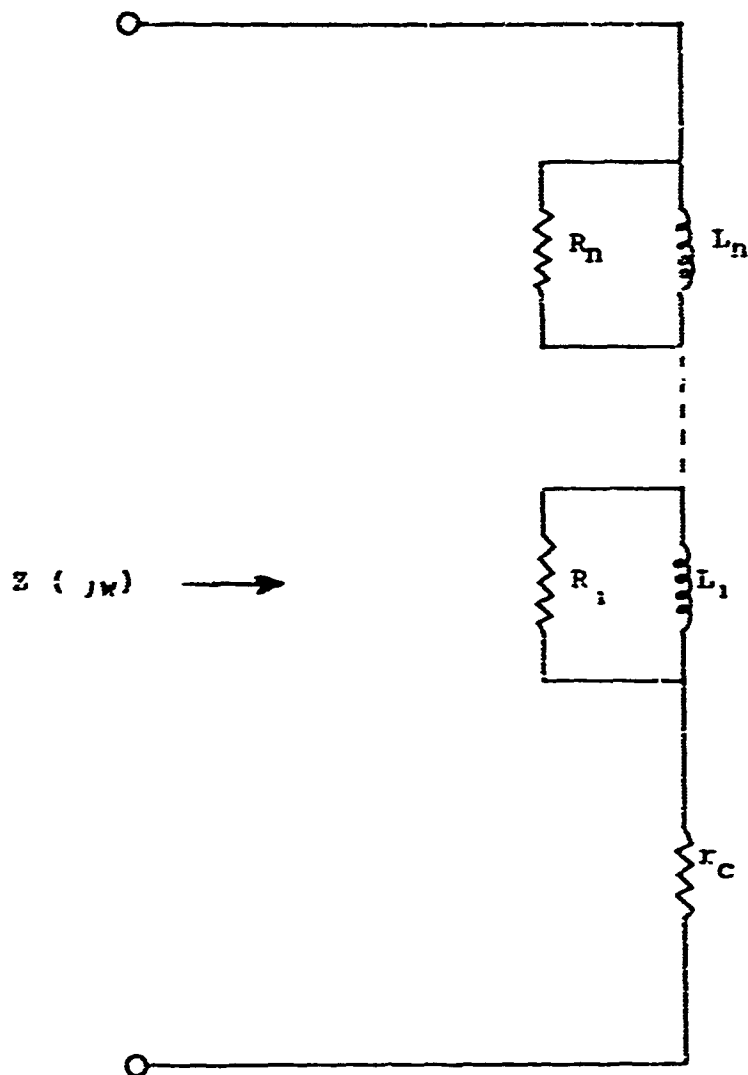


Figure 13-4. Basic Equivalent Circuit for Glow Discharge Tube Operated in Negative-Resistance Region

independent of the plate - cathode spacing and the geometry of the electrodes. It was found to be proportional to the current through the terminals of the tube. Therefore, from experimental results, a large portion of the effective capacitance seems to be caused by the discharge process in the cathode fall region.

Measurements revealed that the effective resistance of the glow discharge tube is inversely proportional to the current through the terminals of the tube at a constant pressure. This resistance was found to increase with plate - cathode separation and with reduction of plate size. Consequently, a large part of the effective resistance is associated with plate - cathode spacing, plate size, and the current through the terminals.

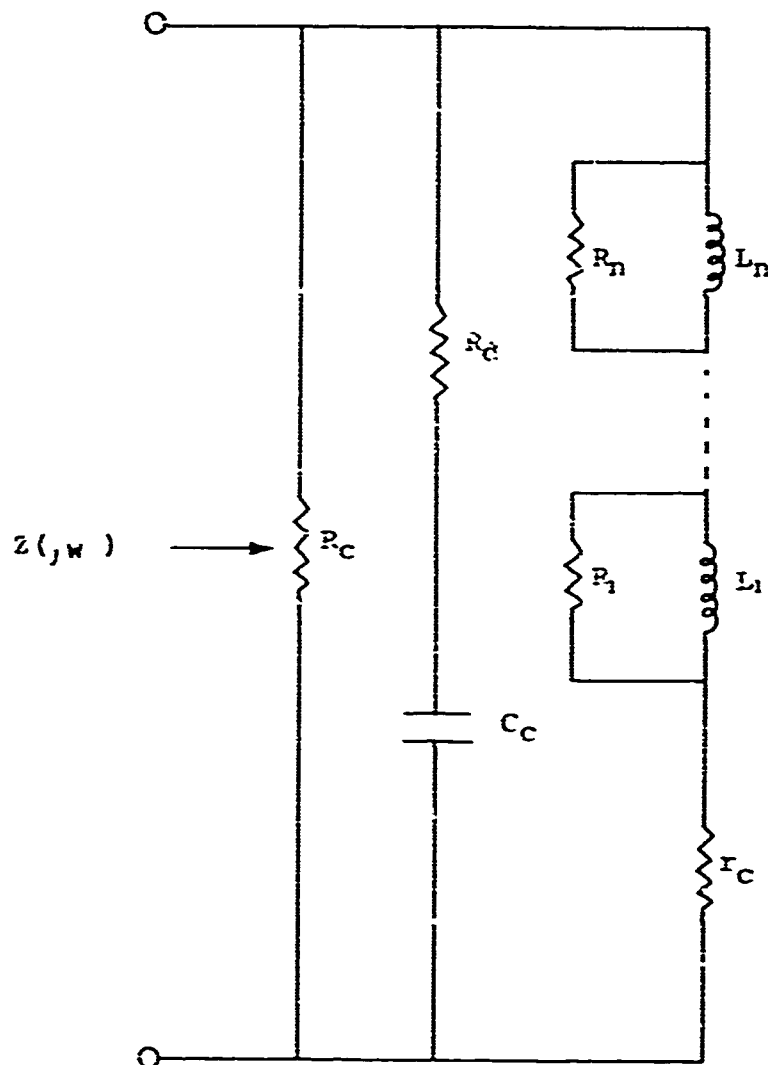


Figure 13-5. Actual Equivalent Circuit for Glow Discharge Tube Operated in Negative-Resistance Region

The load resistance  $R_C$  is the internal resistance of the source and is usually very large.

The incremental resistance  $r_C$  is negative since the glow discharge tube is biased in the negative slope portion of the current/voltage characteristic curve. The reciprocal of the slope of the characteristic curve is equal to the incremental resistance. From the equivalent circuit as shown in Figure 13-4, it can be seen that when the frequency is equal to zero, the input impedance is approximately equal to the incremental resistance

$$Z_C(j\omega) \approx r_C$$



Most of the time constants are relatively independent of the current through the terminals of the tube for the glow discharge. The inductor and the resistor associated with the respective time constants are inversely proportional to the current through the terminals. Since  $L_n/R_n = \tau_n$  and  $L_n$  as well as  $R_n$  are proportional to  $1/i_c$ ,  $\tau_n$  is equal to a constant.

There are five time constants to consider in the glow discharge. The first and most important of the delayed effects  $\tau_1$  is associated with the time taken for the positive ions to arrive at the cathode from their respective creation sites in the cathode fall region. The delay is due to the small mobility of the positive ions before the degree of ionization reaches a new value. The ion transit time is dependent on the current through the terminals of the tube since the degree of ionization in the cathode fall changes with the current through the terminals. The inductance  $L_1$  which is associated with the impedance of the positive ions is related to the nature of the conduction in the cathode fall. The resistance  $R_1$  which is also associated with the impedance of the positive ions should be greater than the resistances associated with other processes, since the ions are the principal current carriers in the cathode fall. The measured value of the ion transit time  $\tau_1$ , is found to be approximately  $0.025 \mu$  seconds.

The larger time constants which are related to processes in the discharge are relatively independent of the current through the terminals of the tube. The second of the delayed effects  $\tau_2$  is associated with the time required for the glow to expand over the cathode surface. The value of time constant  $\tau_2$  is found to be approximately  $200 \mu$  seconds. The third of the delayed effects  $\tau_3$  is related to the time for the anode space charge to change. The value of the time delay  $\tau_3$  is found to be approximately  $15 \mu$  seconds. The fourth of the delayed effects is associated with the time taken for metastable atoms to reach the cathode and liberate electrons. The value of the time constant  $\tau_4$  is found to be approximately  $2 \mu$  seconds. The fifth of the delayed effects is related to the time taken for photons to reach the cathode and liberate electrons. The value of the time constant  $\tau_5$  is found to be approximately  $0.4 \mu$  seconds.

It should be noticed that the actual equivalent circuit for the glow discharge tube which is operated in the negative resistance region as shown in Figure 13-5 resembles the complete equivalent circuit of the current-controlled negative-resistance device as shown in Figure 4-4. These two equivalent circuits will be exactly identical if the following modifications are made in Figure 13-5.

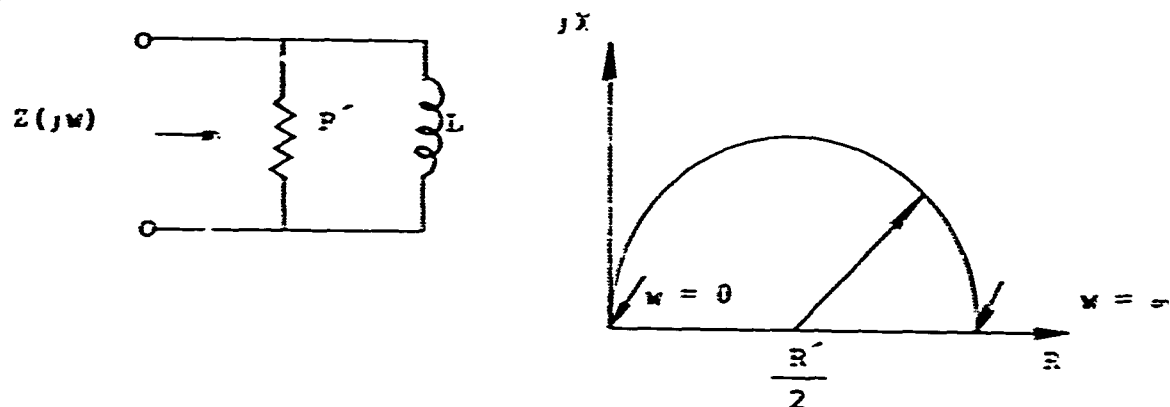
$$R_d = 0$$

$$R_1, R_2, \dots, R_n \rightarrow \infty$$

$$L_1 + L_2 + \dots + L_n = L_c$$

TABLE 13-1. IMPEDANCE LOCUS OF PARALLEL R'L NETWORK

A method of representing a network function involves plotting the real part of the function against the imaginary part. This concept is illustrated in terms of the input impedance of the parallel R'L network.



$$Z(j\omega) = \frac{\omega^2 L^2 R'}{R'^2 + \omega^2 L^2} + j \frac{\omega L R'^2}{R'^2 + \omega^2 L^2}$$

$$R = \frac{\omega^2 L^2 R'}{R'^2 + \omega^2 L^2} \quad X = \frac{\omega L R'^2}{R'^2 + \omega^2 L^2}$$

$$\frac{R}{X} = \frac{\omega L}{R'}$$

$$R^2 - R' R + X^2 = 0$$

$$\left[ R - \frac{R'}{2} \right]^2 + X^2 = 0$$

## SECTION XIV

### THEORY FOR THE VOLTAGE-CONTROLLED NEGATIVE-RESISTANCE DEVICE USING THE TETRODE VACUUM TUBE FOR AN EXAMPLE

In the tetrode as shown in Figure 14-1, the main purpose of the plate is to collect electrons which succeed in passing through the screen. The total space charge current  $i_T$  is almost constant with fixed control-grid and screen-grid voltages. Consequently, the portion of the space charge current which is not collected by the plate must be collected by the screen. It can be seen from Figure 14-1 that the sum of the plate and screen-grid currents is approximately equal to the total space charge current, when the control-grid current is negligible.

$$i_T \approx i_p + i_s$$

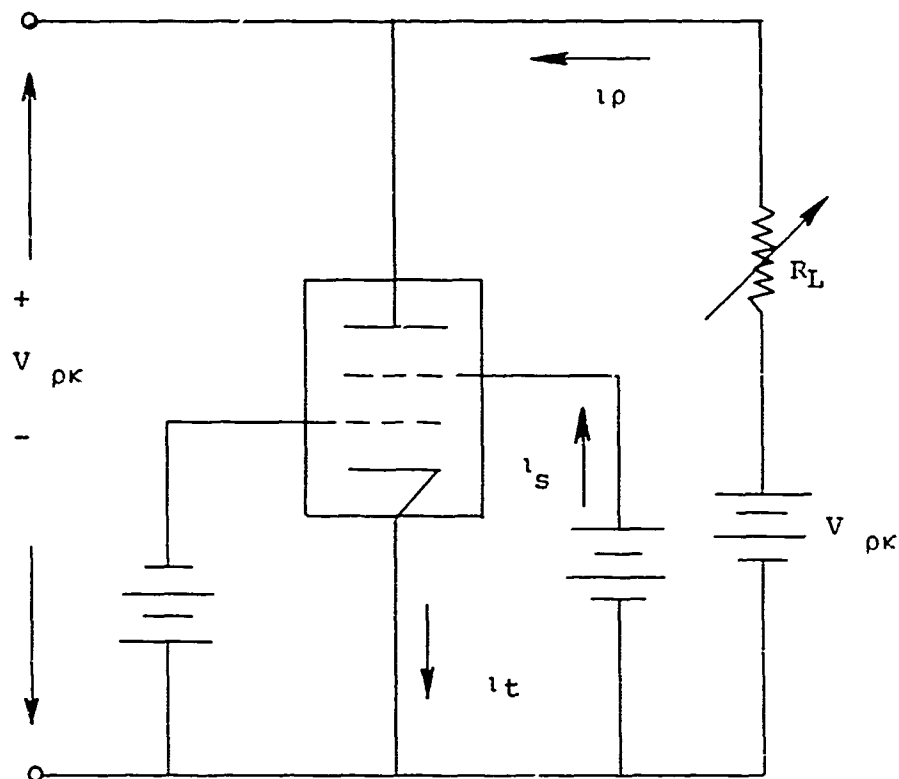


Figure 14-1. Schematic Circuit of a Tetrode

The theory for the voltage-controlled negative-resistance device (vacuum tube tetrode) is discussed by Chirlan and Zemanian (10), Milman and Seely (21) as well as Yeh and Chaffee (34).

The static plate voltage  $V_{PK}$  determines the division of the space charge current between the plate and the screen, but the plate voltage does not seriously affect the total space charge current. A negative-slope region occurs in the static current/voltage characteristic curve for the tetrode at the lower plate voltages as shown in Figure 14-2. The static characteristic curve was determined at the plate-cathode terminals for a constant screen voltage. At zero plate voltage, a few of the electrons have sufficient energy to reach the plate. Therefore, the plate current is very small. When the plate voltage is increased, a rapid increase in the plate current happens with a similar decrease in the screen current. When the plate voltage is much greater than the screen voltage, the plate current will almost approach the space charge current. Consequently, the screen current will almost approach zero.

The plate current enlarges very fast with increasing the plate voltage, but this enlargement is succeeded by a region where the plate current decreases with increasing the plate voltage. The tetrode possesses a negative plate resistance in this region. The negative slope in the characteristic curve as shown in Figure 14-2 is caused by the liberation of secondary electrons from the plate by the impact of the primary electrons with the plate. The secondary electrons are attracted to the screen because it

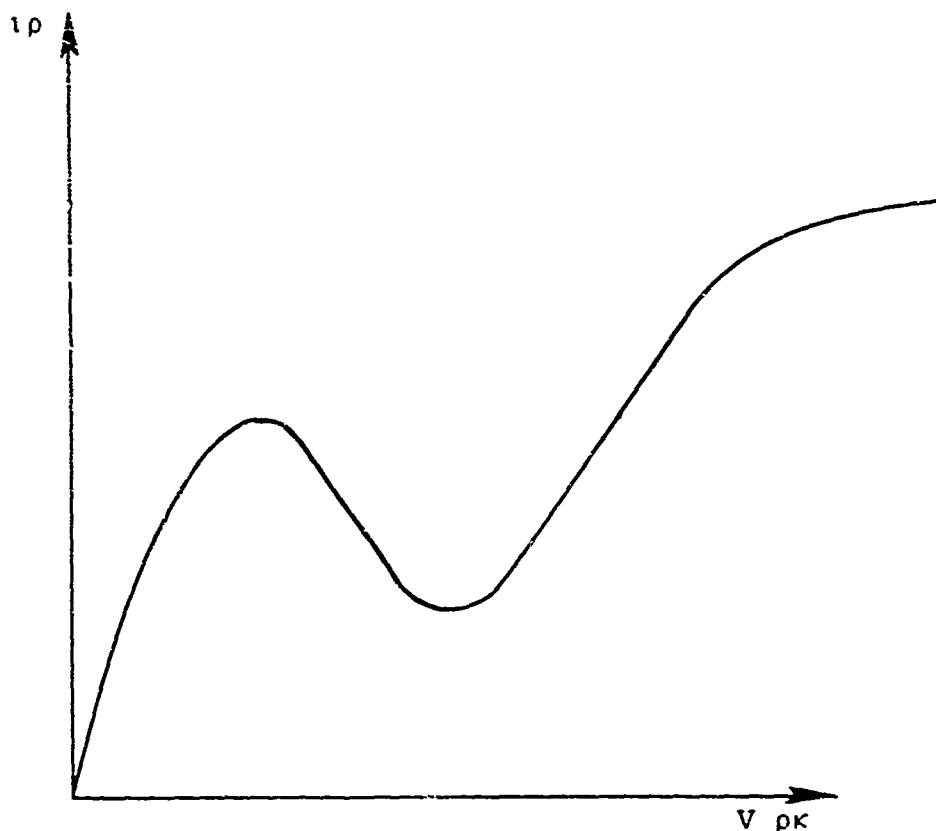


Figure 14-2. Current/Voltage Characteristic Curve for Tetrode Vacuum Tube

is the electrode with the highest positive voltage. The plate current is decreased whereas the screen current is increased in the negative-slope region. The number of secondary electrons that are liberated by the primary electron bombardment depends upon many factors. The number of secondary electrons may exceed the total number of primary electrons that strike the plate. If this occurs, the result is an effective negative plate current.

In the region where the plate voltage is higher than the screen voltage, the secondary electrons which are released from the plate by the collision of the primary electrons are pulled back to the plate. Also the secondary electrons which are liberated from the screen by the impact of primary electrons on it are drawn to the plate. Consequently, the plate current is slightly higher than it would be in the lack of secondary emission from the screen. The plate current persists to increase with increasing plate voltages, because the collection of the secondary electrons is more complete. At the corresponding time, the screen current approximately approaches zero.

In the negative-resistance region, the theoretical input admittance locus of the tetrode at the plate-cathode terminals may have the same form as the input impedance locus of the glow discharge tube when the principle of duality is used. The process of secondary emission in the vacuum tube tetrode in the negative-resistance region may be considered a delayed function of current and voltage because this process may be associated with a delay time. Secondary emission may influence the form of the input admittance locus considerably. Therefore, any process which affects the delay of current as well as voltage and has a time constant associated with it, may be treated as a delayed effect.

The theoretical input admittance of the vacuum tube tetrode at the plate-cathode terminals in the negative-resistance region may have the following form

$$Y(j\omega) = 1/R_0 - \sum_{k=1}^{k=n} \frac{1/R_k}{1 + j\omega\tau_k} \quad (14-1)$$

where  $R_0, R_1, \dots, R_k, \dots, R_n$  are resistances and  $\tau_1, \tau_2, \dots, \tau_k, \dots, \tau_n$  are time constants associated with various delay processes. The second term in equation 14-1 may involve the contributions from the secondary processes of electron production. The quantity  $R_0$  in equation 14-1 is given by

$$\frac{1}{R_0} = \frac{1}{r_v} + \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$

where  $1/r_v$  is the slope of the static current/voltage characteristic curve.

Therefore, the theoretical input admittance of the tetrode at the plate-cathode terminals for  $n$ -delayed effects can be written

$$Y(j\omega) = \frac{1}{r_v} + \frac{j\omega C_1}{1 + j\omega C_1 R_1} + \dots + \frac{j\omega C_n}{1 + j\omega C_n R_n} \quad (14-2)$$

where  $C_1, C_2, \dots, C_n$  are capacitances which are associated with the various delay processes and

$$R_n C_n = \tau_n$$

The second and subsequent terms of equation 14-2 represent a series combination of a capacitor  $C_n$  and a resistor  $R_n$ . The basic equivalent circuit can be associated with the form of the theoretical input admittance as shown in Figure 14-3. It can be represented by a parallel combination of the incremental resistance and a number of series combinations of capacitors and resistors. The actual equivalent circuit as

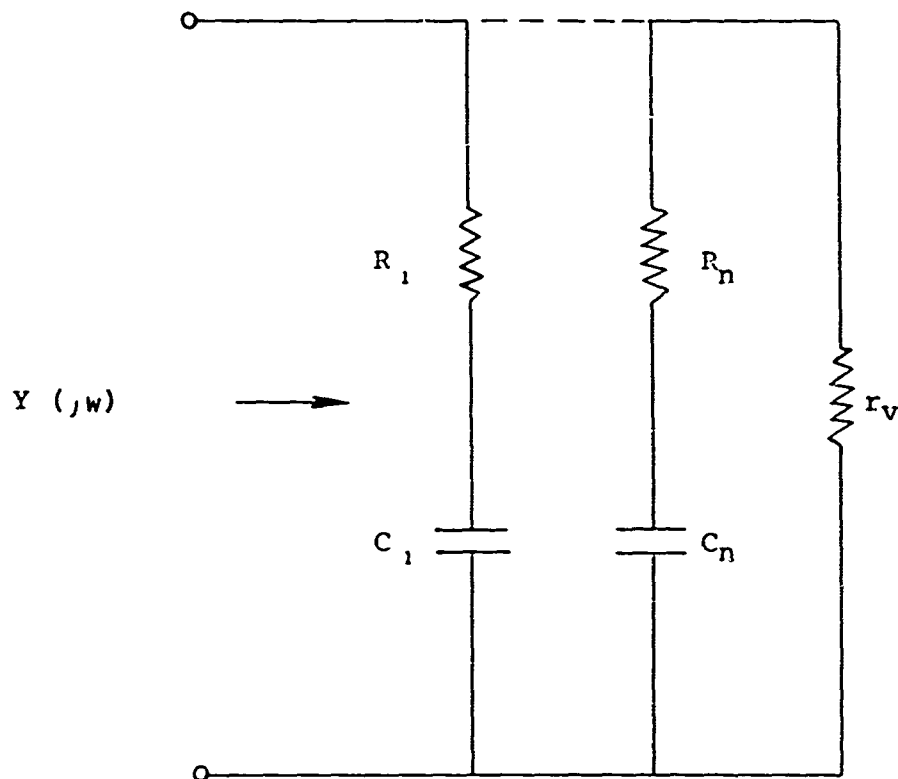


Figure 14-3. Basic Equivalent Circuit for Tetrode Vacuum Tube Operated in Negative-Resistance Region

shown in Figure 14-4 also contains the effective inductance  $L_v$ , the effective resistance  $R_s$  and the load resistance  $R_v$ . Each series combination of  $C_n$  and  $R_n$  has a time constant which is associated with a particular secondary process of electron production.

Experiments show that the vacuum tube tetrode which is operated in the negative plate resistance region can be used as an oscillator. Therefore, an effective inductance and an effective resistance are involved in the actual input admittance of the tetrode at the plate-cathode terminals. A large part of the effective resistance and the effective inductance may be due to the leads and the vacuum tube housing.

The load resistance  $R_v$  is the internal resistance of the source and is usually very small.

The incremental plate resistance  $r_v$  is negative because the tetrode is biased in the negative slope portion of the current/voltage characteristic curve. The reciprocal of the slope of the characteristic curve is equal to the incremental resistance. From

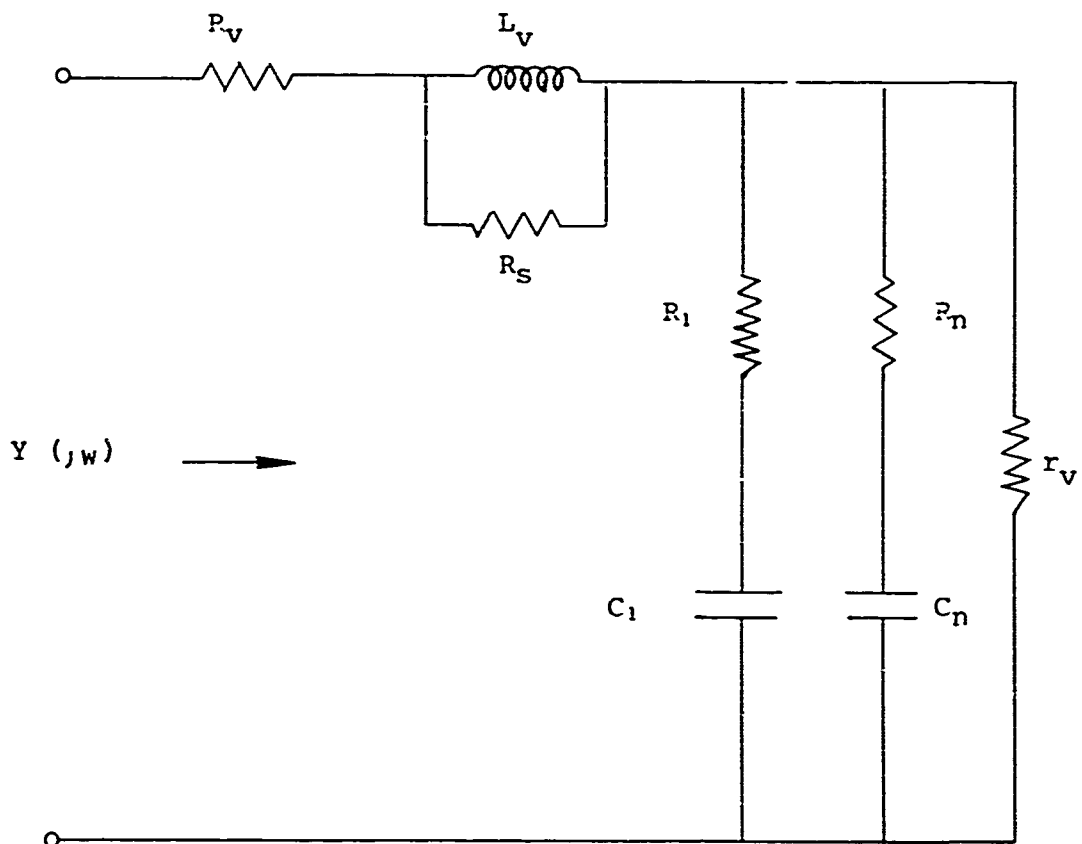


Figure 14-4. Actual Equivalent Circuit for Tetrode Vacuum Tube Operated in Negative-Resistance Region

the equivalent circuit as shown in Figure 14-4, it can be seen that when the frequency is equal to zero, the input admittance is approximately equal to the reciprocal of the incremental resistance  $Y(j\omega) \approx 1/r_v$ .

There may be several time constants to consider when the tetrode is operating in the negative-resistance region. Probably the most important of the delayed effects may be identified with the time taken for the primary electrons to reach the plate and to liberate secondary electrons which are collected at the screen. The delay may be due to the mobility of the electrons before the degree of secondary emission reaches a new value. The electron transit time may be dependent on the voltage across the plate-cathode terminals of the tube, because the degree of secondary emission changes with the voltage between the terminals. The capacitance  $C_1$  which may be identified with the admittance of the secondary emission mechanism may be associated with the behavior of the conduction in the secondary emission process. The resistance  $R_1$  which is also identified with admittance of the secondary emission mechanism should be very small because it is assumed that the electron transit time is extremely small.

It should be observed that the actual equivalent circuit for the tetrode vacuum tube that is operated in the negative resistance region as shown in Figure 14-4 is similar to the complete equivalent circuit for the voltage-controlled negative-resistance device as shown in Figure 4-8. These two equivalent circuits will be precisely the same if the succeeding changes are made in Figure 14-4.

$$R_s \rightarrow \infty$$

$$R_1, R_2, \dots, R_n = 0$$

$$C_1 + C_2 + \dots + C_n = C_v$$



## SECTION XV

### LIST OF NEGATIVE-RESISTANCE DEVICES

Simple negative-resistance devices are those devices which exhibit the negative resistance and the control of the phenomenon that causes the negative resistance at the same pair of terminals. These simple devices are either current-controlled or voltage-controlled. A few of the simple negative-resistance devices are listed below.

1. Glow Discharge Diode Tube and Arc Discharge Diode Tube, Current Controlled, Figure 15-1.

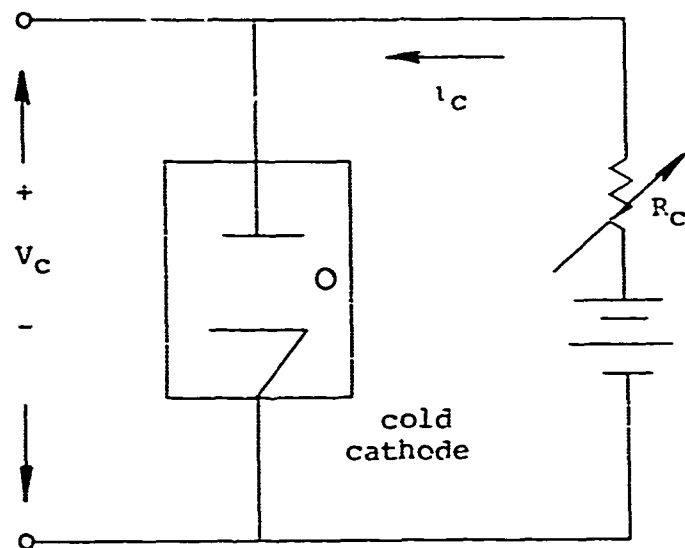
The operation of the arc discharge is extremely analogous to the glow discharge. The dissimilarity between the arc discharge and the glow discharge is that the arc discharge is sustained with a higher current and a lower voltage. For a certain range of values, an increasing current through the terminals of a glow or arc discharge diode tube causes an increase in the ionization. The result is that a smaller voltage through the terminals is needed to maintain the discharge. The slow positive ion mobility creates a time lag between the current and the voltage when the positive ions pass through the cathode fall region.

2. Tetrode Vacuum Tube and Dynatron Vacuum Tube, Voltage Controlled, Figure 15-2.

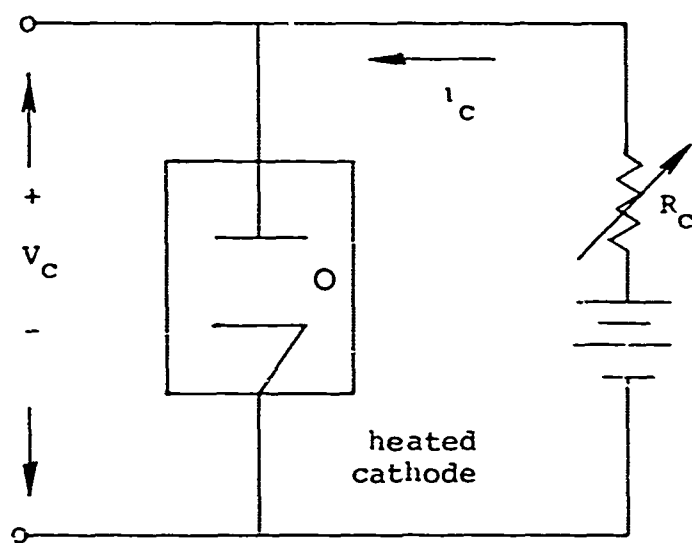
The behavior of the dynatron is very similar to the tetrode. The difference between them is that the dynatron does not have a control grid. Over a particular range of values an increasing plate voltage for a constant screen voltage brings about an increase in the secondary emission of electrons from the plate. When the number of secondary electrons which are collected by the screen exceed the number of primary electrons that strike the plate, the plate current decreases. The electron mobility may produce a time delay between the voltage and the current when the secondary electrons travel from the plate to the screen.

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A great many books and papers exist which explain the operation of various kinds of negative-resistance devices. Some of these are Culwell, Devletoglou, and Niemann (11), Hull (16), Lesk and Mathis (17), Lynn, Pepper and Pederson (18), Misawa (23), Nanavati (25), Rindner and Schmid (26), Shtager and Stafeyev (28), Vickers (33) as well as the references used in Sections 12 and 13.

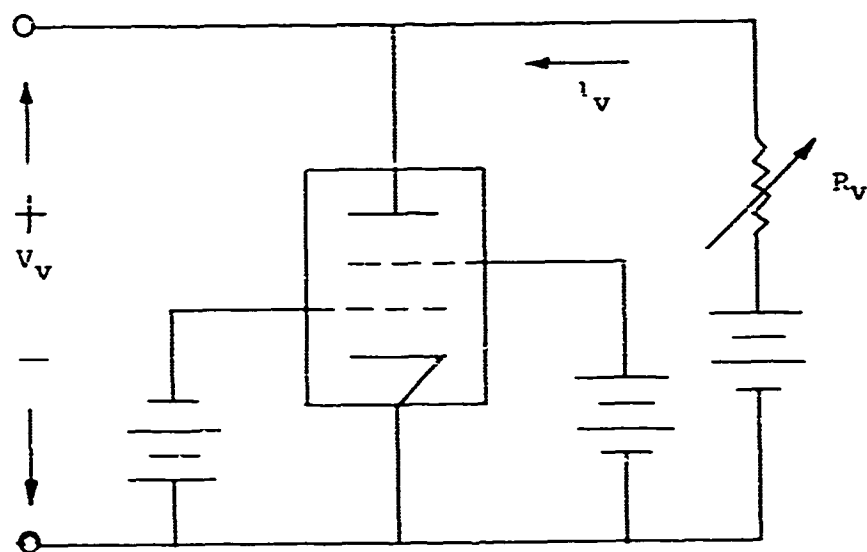


Glow Discharge Diode Tube

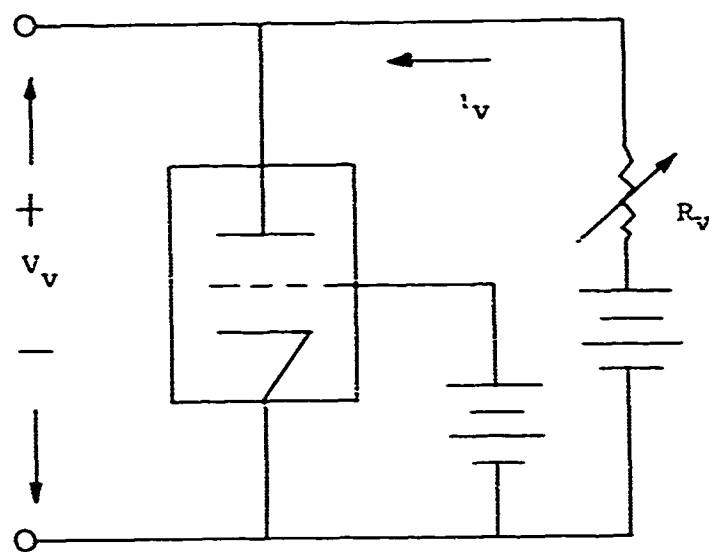


Arc Discharge Diode Tube

Figure 15-1



Tetrode Vacuum Tube



Dynatron Vacuum Tube

Figure 15-2

3. Tunnel Diode, Voltage Controlled, Figure 15-3.

For a specific range of values an increasing voltage between the terminals of the tunnel diode creates an increase in the tunneling current which opposes the normal diffusion current. The effect is that the total current through the terminals decreases. The electron and hole mobilities may cause a time lag between the voltage and the current when the electrons and holes go through the depletion region which is a voltage barrier.

4. Bonded Negative-Resistance Diode and PNP Diode, Current Controlled, Figure 15-4.

The operation of the BNR diode is analogous to the PNP diode. The difference between the BNR diode and the PNP diode is that the BNR diode has two layers whereas the PNP diode has four layers. Over a given range of values an increasing current through the terminals of a BNR diode or a PNP diode brings about a phenomenon called avalanche breakdown that produces a smaller bulk resistance. The result is that a smaller voltage across the terminals is required to maintain a particular current. The electron and hole mobilities may cause a time delay between the current and the voltage when the electrons and holes move through the depletion region.

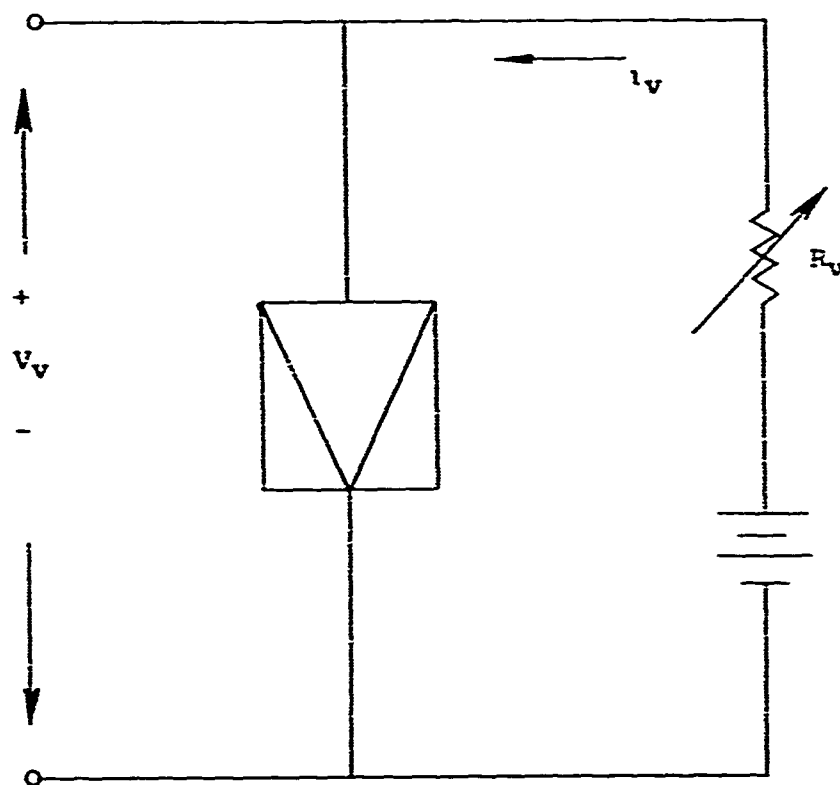
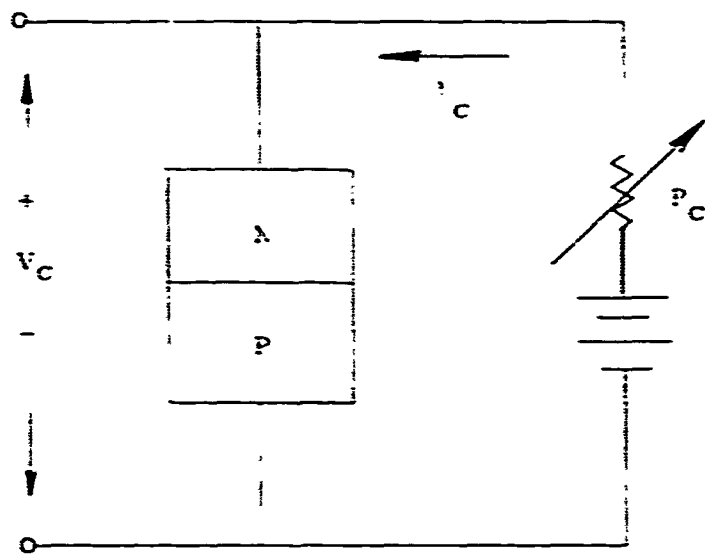
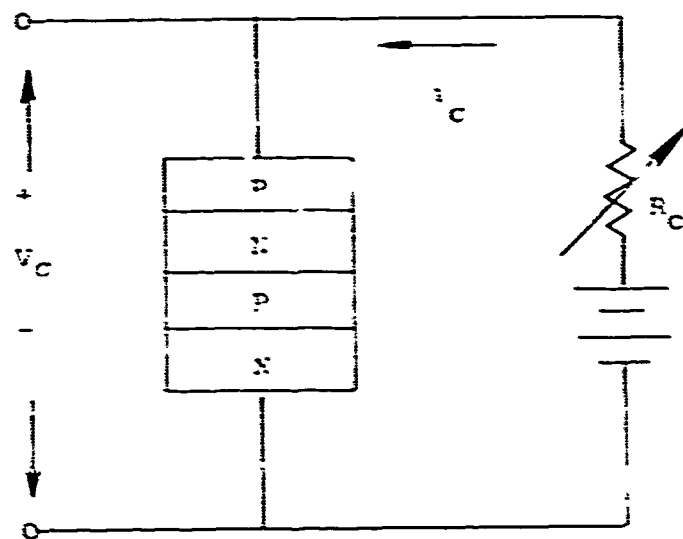


Figure 15-3. Tunnel Diode



Bonded Negative-Resistance Diode



Avalanche Diode

Figure 15-4

5. Unijunction Transistor (Double-Base Diode), Current Controlled, Figure 15-5.

For a certain range of values an increasing current through the emitter - base 1 terminals at a constant base 2 - base 1 voltage of the unijunction transistor causes the bulk resistivity near base 1 to decrease. The effect is that a smaller voltage across the emitter - base 1 terminals is required to maintain the specified current. The electron and hole mobilities may produce a time lag between the current and the voltage when the electrons and holes transport through the depletion region.

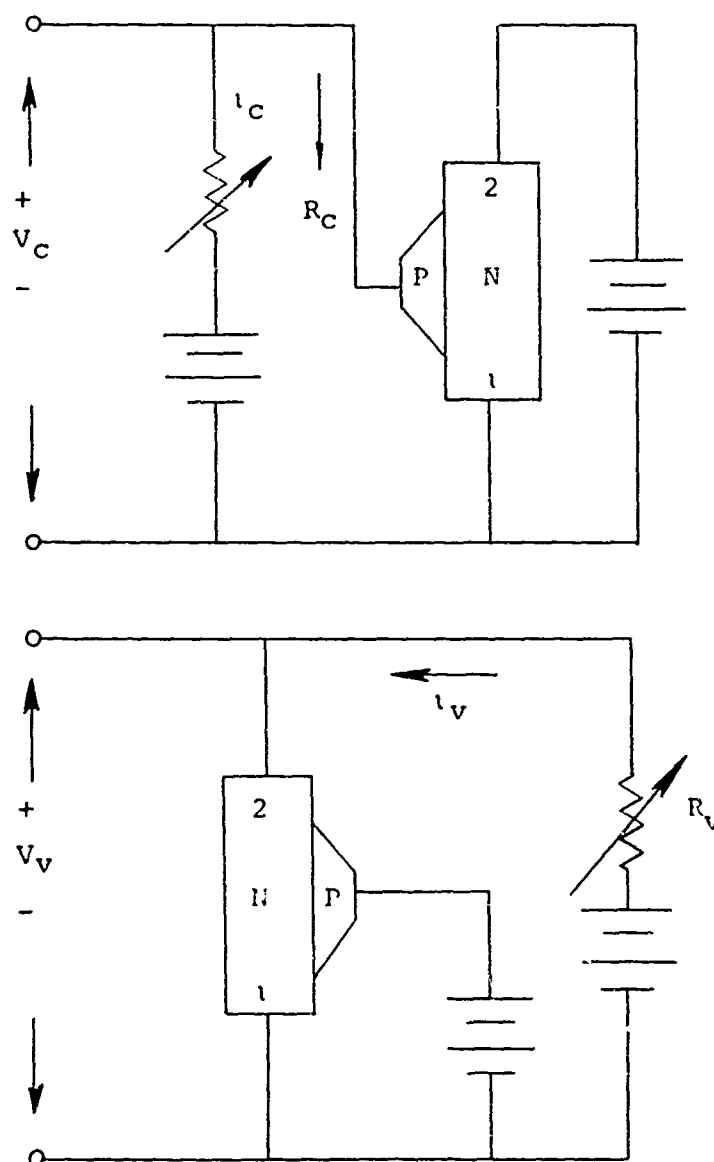


Figure 15-j. Unijunction Transistor

6. Double-Base Diode, Voltage Controlled, Figure 15-5.

Over a particular range of values an increasing voltage across the base 2 - base 1 terminals at a constant emitter - base 1 voltage produces an increase in the bulk conductivity of the base region. The result is that a smaller current flows through the base 2 - base 1 terminals. The electron and hole mobilities may bring about a time delay between the voltage and the current when electrons and holes cross through the depletion region.

This section has shown with some examples that the mechanism which creates the negative resistance is unique for each type of negative resistance device (vacuum tube, gas tube, semi-conductor) selected for illustration.

## APPENDIX TO SECTION XII

### LIMIT CYCLE

The polynomial approximation is used for the non-linear characteristic curve instead of the piece-wise linear approximation. The following equations were derived from Figure 9-1.

$$\frac{dv}{dt} = -\frac{1}{C_c} \left[ i(t) + \frac{v(t)}{R_c} \right] \quad (9-1)$$

$$\frac{di}{dt} = \frac{1}{L_c} \left[ v'(t) - f(i_c) \right] \quad (9-2)$$

A third order polynomial approximation is used for the non-linear current/voltage characteristic curve for the current-controlled negative-resistance device

$$f(i_c) = V'_s - ai + bi^3$$

Equation 9-2 can be written

$$\frac{di}{dt} = \frac{1}{L_c} \left[ v(t) + ai - bi^3 \right] \quad (12-1)$$

The following non-linear, second-order, first-degree differential equation is obtained from equations 9-1 and 12-1.

$$\frac{d^2i}{dt^2} + \left[ \frac{1}{R_c C_c} - \frac{a}{L_c} + \frac{3bi^2}{L_c} \right] \frac{di}{dt} + \left[ \frac{bi^2}{R_c} - \frac{a}{R_c} + 1 \right] \frac{i(t)}{L_c C_c} = 0 \quad (12-2)$$

It is assumed that

$$\left[ \frac{bi^2}{R_c} - \frac{a}{R_c} \right] \ll 1$$

---

A reasonable derivation of the van der Pol equation using the third order approximation for the non-linear current voltage characteristic curve is explained by Mc Lachlan (20). The isocline method of solving the van der Pol equation is discussed by Cunningham (12).



so that equation 12-2 can be written

$$\frac{d^2 i}{dt^2} + \left[ \frac{1}{R_c C_c} - \frac{a}{L_c} + \frac{3bi^2}{L_c} \right] \frac{di}{dt} + \frac{i(t)}{L_c C_c} = 0 \quad (12-3)$$

$$\text{Let } B = \left[ \frac{a}{L_c} - \frac{1}{R_c C_c} \right]$$

$$c = \frac{3b}{L_c}$$

$$\alpha = \frac{1}{L_c C_c}$$

Equation 12-3 can be written

$$\frac{d^2 i}{dt^2} + [-B + ci^2] \frac{di}{dt} + \alpha i(t) = 0 \quad (12-4)$$

$$\text{Let } i(t) = \left[ \frac{B}{c} \right]^{\frac{1}{2}} x(t)$$

Equation 12-4 has the form

$$\frac{d^2 x}{dt^2} - B[1 - x^2] \frac{dx}{dt} + \alpha x(t) = 0 \quad (12-5)$$

$$\text{Let } t = \alpha^{-\frac{1}{2}} \tau$$

$$\epsilon = \alpha^{-\frac{1}{2}} B$$

The following non linear van der Poi equation is obtained.

$$\frac{d^2 x}{d\tau^2} - \epsilon [1 - x^2] \frac{dx}{d\tau} + x(\tau) = 0 \quad (12-6)$$

This is the equation for an oscillatory system which has variable damping. If the displacement  $x$  is small, the damping is negative; however, if the displacement  $x$  is large, the damping is positive. Equation 12-6 describes the behavior of negative

resistance devices which are used as oscillators. The nature of the solution of equation 12-6 depends on the value of the parameter  $\epsilon$ . If  $\epsilon < 1$ , one type of solution results and if  $\epsilon > 1$ , another type of solution results.

This isocline method is used to find the phase plane solution for the van der Pol equation.

$$\text{Let } y = \frac{dx}{d\tau}$$

Equation 12-6 can be written

$$\frac{dy}{dx} = \frac{\epsilon[1 - x^2]y - x}{y} \quad (12-7)$$

The algebraic equation for an isocline curve is

$$y = \frac{x}{\epsilon[1 - x^2] - m} \quad (12-8)$$

where  $m$  is a specified value of the slope  $dy/dx$  of the solution curve. Isocline curves from equation 12-8 with certain values of  $m$  are plotted in Figure 12-4 for a specified value of  $\epsilon$ .

The steady state solution curve for the van der Pol equation is closed for any type of initial conditions. This closed curve represents a steady - state periodic oscillation which is determined by the properties of van der Pol equation and not determined by the method that starts the oscillation. If the initial point is inside the limit cycle, the transient solution curve spirals outward toward the limit cycle. Similarly if the initial point is outside the limit cycle, the transient solution curve spirals inward toward the limit cycle.

A similar derivation of the van der Pol equation can be done by considering the following equations which were derived from Figure 9-2.

$$\frac{di}{dt} = -\frac{1}{L_V} [v(t) + i(t)R_V] \quad (9-5)$$

$$\frac{dv}{dt} = \frac{1}{C_V} [i'(t) - f(V_V)] \quad (12-9)$$

A third order polynomial approximation is used for the non-linear current/voltage characteristic curve for the voltage-controlled negative-resistance device

$$f(v_V) = 1'_S - av + bv^3$$

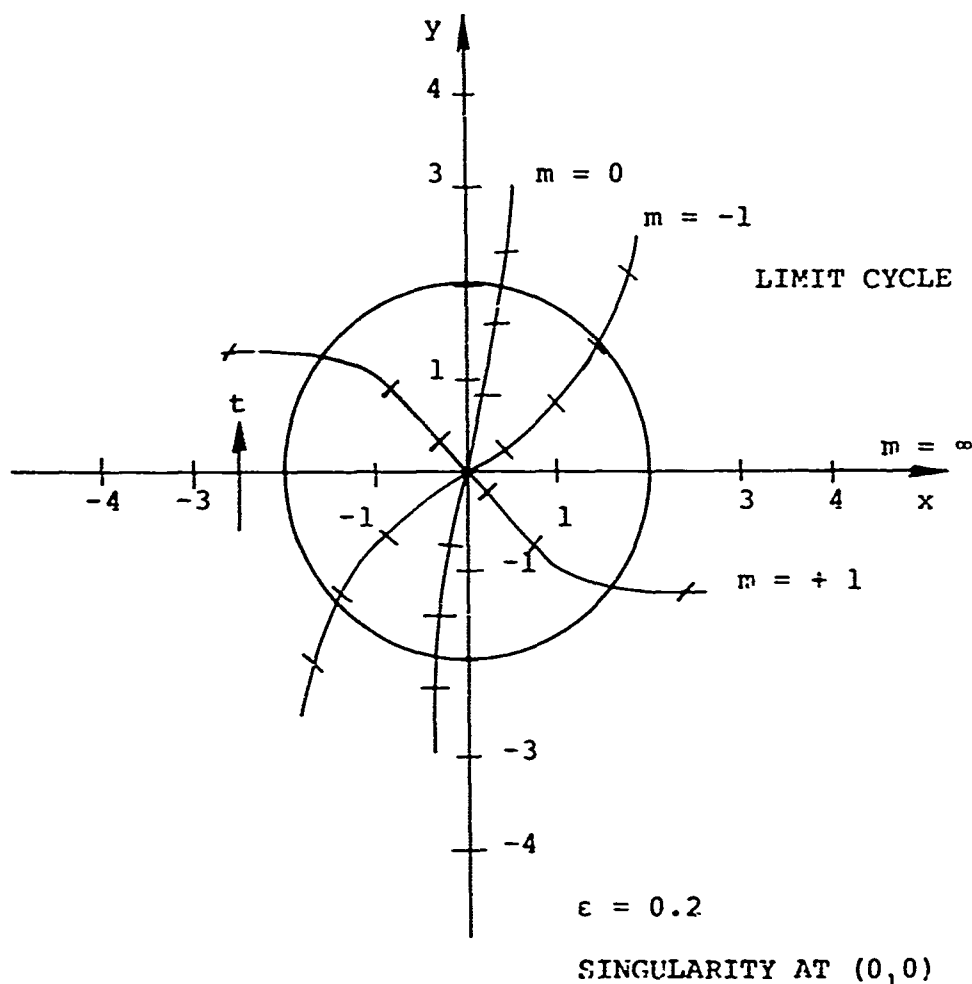


Figure 12-4. Phase-Plane Diagram for Van Der Pol Equation with Isocline Construction Curves

The following non-linear differential equation is obtained from equations 9-5 and 12-9.

$$\frac{d^2v}{dt^2} + \left[ \frac{R_v}{L_v} - \frac{a}{C_v} + \frac{3bv^2}{C_v} \right] \frac{dv}{dt} + \left[ bR_v v^2 - aR_v + 1 \right] \frac{v(t)}{L_v C_v} = 0 \quad (12-10)$$

By assuming

$$\left[ bR_v v^2 - aR_v \right] \ll 1$$

and making several substitutions, equation 12-10 will have the form of the van der Pol equation.

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14. ABSTRACT <p>This report examines certain phases of the dynamic behavior of an open-circuit stable, two-terminal negative-resistance device with its specific equivalent circuit, as well as the dynamic behavior of a short-circuit stable, two-terminal negative-resistance device with its specific equivalent circuit. The equivalent circuits for these two devices are duals of each other. The static current/voltage characteristic curves for the two types of negative-resistance devices affect their dynamic equivalent circuits.</p> <p>It is shown that the static current/voltage characteristic curve for the negative-resistance device has a limited negative-slope region since the device possesses particular physical restrictions. The complete equivalent circuit for the current-controlled and voltage-controlled negative-resistance devices is derived by recognizing specific information about the device. The basic equivalent circuit for the negative-resistance device is obtained by assuming a cause-effect relationship between the current and voltage of the device and then by using the Taylor series expansion on the cause. The basic equivalent circuit for the negative-resistance device is related to the static current/voltage characteristic curve when the device is operated as a trigger circuit. Expressions for the input admittance of the current-controlled negative-resistance device and the input impedance of the voltage-controlled negative-resistance device are derived from the equivalent circuit of the device. The homogeneous linear second-order differential equation that describes the limited behavior of the negative-resistance device is also obtained.</p>			

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	Abstract (continued)  from the equivalent circuit of the device. The fundamental energy storage element is determined for the basic equivalent circuit to approximate the behavior of the negative-resistance device. The stability regions which come from the homogeneous linear second-order differential equation pertain to the operation of the negative-resistance device.  When the negative-resistance device is operated as an oscillator, the instantaneous operating point describes a limit cycle for steady state conditions. The theory of the current-controlled negative-resistance device is explained using the glow discharge tube as an example. The theory of the voltage-controlled negative-resistance device is partially explained and partially simulated when the vacuum tube tetrode is used for an example.						

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